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ABSTRACT

This guide contains a collection of alternative techniques for teaching arithmetic that have proved useful in teaching both children and adults with various learning problems. Discussed first are several underlying principles of instructing individuals with learning disabilities, problems, and differences and special accommodations for learning-disabled students, including the use of graph paper and calculators. The remainder of the guide consists of sections of alternative instructional techniques and exercises dealing with the following topics: odd versus even, reading large numbers, number facts, addition, subtraction, multiplication, division, fractions, decimals, percentages, measuring, and numbers and shapes. Also included in each section is information about special difficulties that learning-disabled students frequently encounter when learning the specific mathematical concept/operation under consideration and strategies for overcoming those difficulties. (MN)

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Alternative Math Techniques Instructional Guide

by

Richard Cooper, Ph.D.

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Introduction

I have found this collection of alternative techniques for teaching arithmetic useful for teaching both children and adults with a variety of learning problems. Not every technique is necessary for each student to learn, but the more ways a student can manipulate numbers the more likely that student is to remember some way to solve math problems. There are a number of underlying principles which I use to help individuals with learning disabilities, problems and differences.

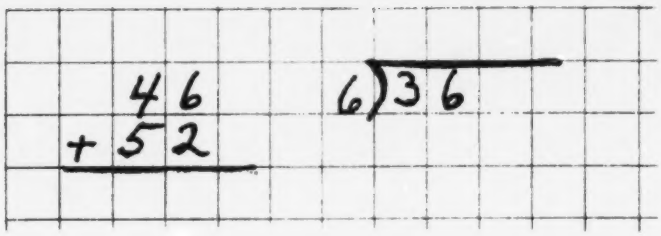
1. Math does not come naturally to everyone.
2. Perceptual, processing and communication differences can effect the learning of math.
3. A person who sequences (counts) rather than groups numbers has difficulty learning math.
4. Individuals with learning problems often need far more repetition than other students.
5. Individuals who do not pay attention to numbers and quantitative concepts in their environments find learning math similar to learning a foreign language. It is very difficult for them, because it is not the way they normally think.
6. Informational gaps limit students' ability to make progress in learning math. Students who have gaps in their understanding of math are often unable to learn higher level operations and concepts, because they do not know a lower level concept or possess a lower level skill. The most obvious example is long division. Students who do not know all the multiplication facts have difficulty mastering division.
7. "If you don't use it, you lose it." This cliche applies to the learning of math. Students who do not use the math skills they learn will, over time, forget them.

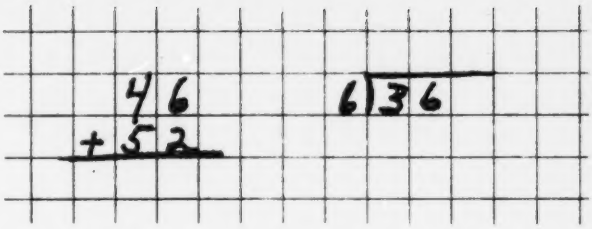
I hope you enjoy learning and using the techniques in this book. If there are other techniques you have found useful, you can send them to me and I will consider including them in future editions of this book.

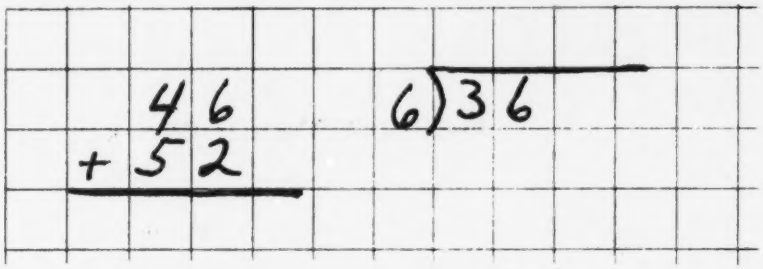
Accommodations

Some students need remediation, others need accommodations. Remediation is the re-instruction of subject matter because it was not mastered when it was originally taught or it was forgotten. An accommodation is an alternative way, or an assistive device, which allows an individual to perform a task which the person otherwise would not be able to perform. Very often both remediation and accommodations are needed to enable students with learning differences to learn and perform mathematical operations. The following are a number of accommodations which can be given to students who have learning problems. These suggestions are aimed at individuals who have specific learning differences, but many of them can be used by all students.

Graph Paper: In some countries, all students use notebooks for math with both vertical and horizontal lines instead of just horizontal lines. In the United States, graph paper is usually only used for certain math procedures such as graphing. Students with visual processing problems should always use graph paper to assist them with lining up numbers. Also, they can see that they must fill a space with a zero when completing operations such as multiplication and division which require place value holders. By using the graph paper for all math activities, most students, even those with significant difficulties with visual organization or motor problems, will be able to line up math problems correctly and keep the work more organized. The use of graph paper should not be seen as an accommodation which is only used when doing homework or completing a test. It should be always be used when the student is doing math.

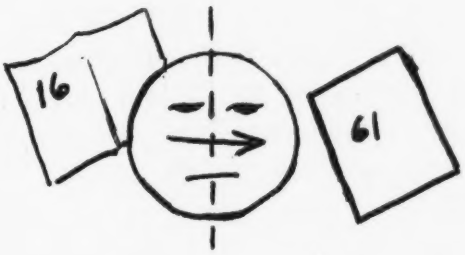




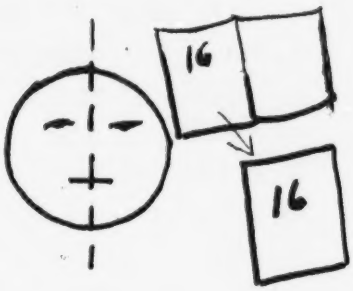


The size of the blocks should match the size of the student's handwriting.

Individuals with right/left discrimination problems often make reversals or transpositions such as writing 61 for 16. One way to reduce these reversals is to eliminate crossing from left to right when copying numbers.



Crossing from left to right



Copying from above to eliminate crossing from left to right.

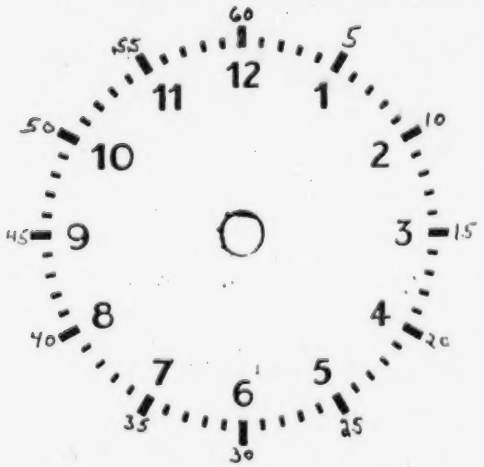
Another accommodation for students with visual processing problems is large print. If large print books are not readily available, then pages can be enlarged on copiers which have enlarging features. If such equipment is not available, then math problems can be rewritten larger on graph paper.

Practice Add or subtract.

1. $8.54 + 0.23	2. $5.42 + 4.68	3. $0.98 + 0.49
4. $5.99 + 0.17	5. $79.18 + 32.82	6. $43.49 + 24.77

Enlarging copy machines can make math problems easier to read and complete.

Telling time is becoming less of a problem for individuals with learning problems because of digital watches and clocks, but reading clocks with hands continues to be a problem for many people. I have found that telling time on clocks with hands is much easier to learn and to use when the student can see the sixty minutes as real numbers rather than as lines or spaces. Try using the clock faces below to teach telling time.



Practice Clock Faces

3

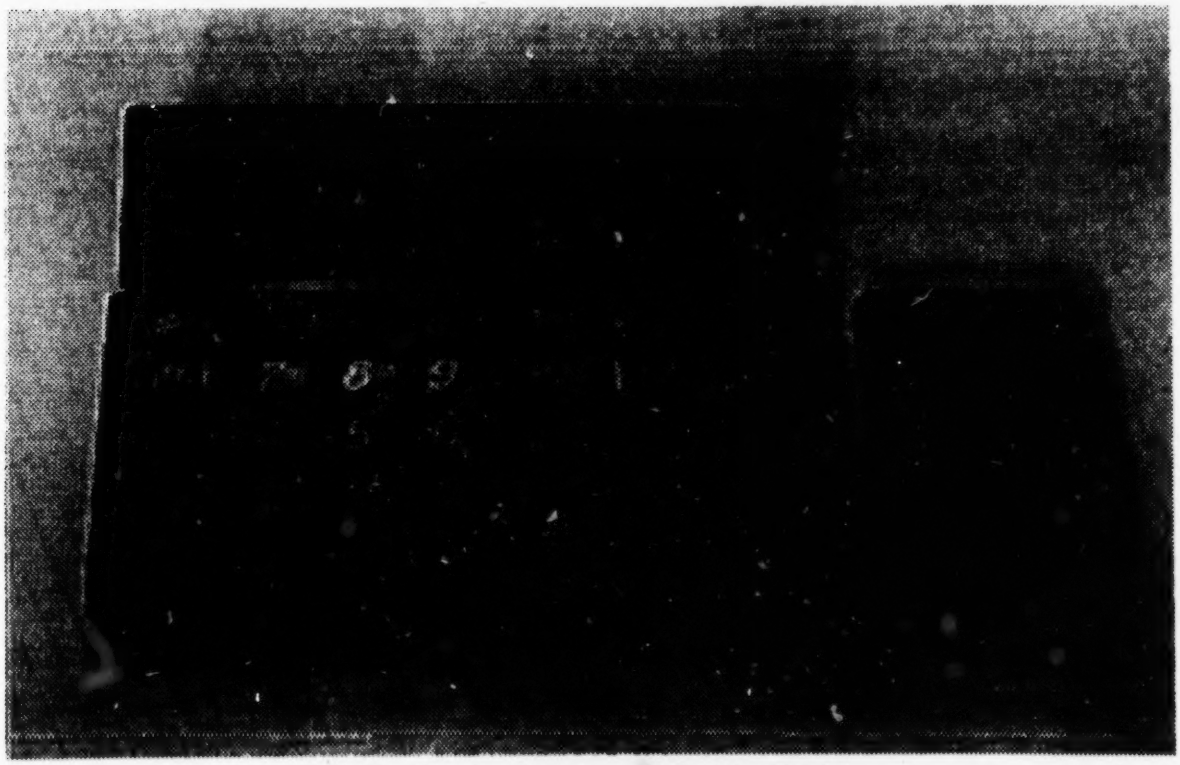
Making change is frequently a problem for students with weak math skills. Many adults believe that they just cannot make change, because they have difficulty with subtraction. Since making change was usually demonstrated in word problems using subtraction, many students view making change as a task which is beyond their ability. Few have been taught to add up to make change. An example of making change with addition is the purchase of an item for $0.29 with a five dollar bill. The student adds the cost of the item and the change until he or she arrives at five dollars. Add $0.29 + one penny and two dimes to arrive at fifty cents. Add two more quarters to make a dollar, and four more dollars brings the total to five dollars. With the item costing $0.29 in one hand and $4.71 in the other hand, the student is able to see that making change is easy when one adds instead of subtracts.

$5.00	$0.29
- .29	+.01
$4.71	.30
	+.10
	+.10
	.50
	+.25
	+.25
	1.00
	+4.00
	5.00
Using subtraction	*Using Addition*

Calculators: There is much discussion these days about the use of calculators as a substitute for learning basic math skills. Students who are good with math will naturally learn and understand the basic concepts and number facts whether they use calculators or not. Student with weak math skills usually have difficulty learning the concept and have even more difficulty if they do not learn the basic number facts and the operations of

arithmetic. A middle of the road approach is probably the most useful for students with learning problems. These students should be taught the basic number facts for use with simple calculations and how to use a calculator for more complex and larger calculations.

The speaking calculators, which pronounce each number as it is entered and the results of calculations, are excellent assistive devices for students who need auditory reinforcement to ensure that they have entered the correct numbers. Other students find that they need to see what they have entered. These students should use calculators with large characters, multi-line display screens or a paper printout. One other consideration is the number of functions which the calculator can perform. As a general rule, students should use the simplest calculator available for the level of math they are learning or using. This provides the student with ease of operation and reduces the chance for touching the wrong keys when entering numbers and operations.



Students with visual or motor difficulties should use calculators with large print and large buttons.

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Odd and Even

Weighted Learning: Because of right/left discrimination problems many students have difficulty remembering the odd and even numbers. Since the odd and even numbers have an "either/or" relationship, students who make reversals find the odd and even numbers confusing. By weighting one side in an "either/or" relationship, the student is able to distinguish between the two sides. This is accomplished by weighting the even numbers. The student learns only the even numbers -- 2, 4, 6, 8 and 10, then if a number is not even, it is odd.

Even
2
4
6 other numbers are odd
8
10

Weighted learning

Another way to help students remember odd and even numbers is by expanding their understanding of the word "odd". Many students with learning problems have an ambiguous vocabulary. They do not fully understand the meaning of many common words. Since the word "odd" is most often used to refer to the strange and weird, students with an ambiguous vocabulary have difficulty understanding what is "odd" or weird about the number 1 or the other odd numbers. Since the idea of a number being weird does not make sense, the student finds odd and even to be an incomprehensible concept. By explaining even as a pair, then one is not a pair, and three is one pair with an odd one. Therefore, the student can better understand the concept of "odd" and it is no longer a confusing term.

Even Pairs and Uneven Pairs

Using Odd and Even for Remembering Number Facts

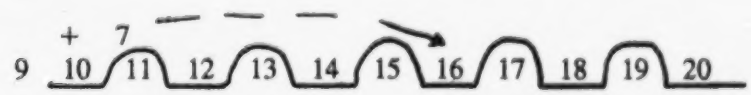
Students can learn to remember the addition and subtraction number facts by using the concept of odd and even.

1. The combination of two even numbers always results in an even number
2. The combination of two odd numbers always results in an even number
3. The combination of an odd and an even number always results in an odd number.

The following diagram of even and odd number facts provides the person with a mathematical concept which will help later when the same pattern is used for the multiplication of signed numbers.

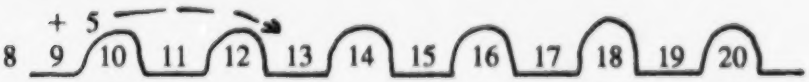
E	O	E	O
+E	+ O	+O	+E
E	E	O	O

For example 9 + 7. Since both are odd numbers, the combination will result in an even number. The student can envision a number line that looks like this.



The student can visualize the even numbers as possible answers. 10 and 12 are too close to 9. 7 + 7 = 14 and 9 + 9 = 18, so the answer is 16.

8 + 5 will produce an odd number. The student would envision the following number line and find that the answer is 13.



Reading Large Numbers

Many students with weak math skills are not able to read large numbers. They were often overwhelmed with the names of place values or have forgotten the terms. Teaching students to read large numbers is easy for most when the following method is used.

First point out the repeating pattern in the large numbers.

Hundred Ten and One, Hundred Ten and One, Hundred Ten and One, Hundred Ten and One

B M T

Billion Million Thousand

HTO, HTO, HTO, HTO, HTO

T B M T

Next, the teacher explains the following diagram and demonstrates how easy it is to read large numbers by following the patterns of the first three numbers. One. Twenty-one. Three hundred, twenty-one. Four thousand, three hundred, twenty-one. Etc. As the student reads each number the rhythm of the repeating patterns becomes easy to remember.

									1
								2	1
							3	2	1
						4,	3	2	1
					5	4,	3	2	1
				6	5	4,	3	2	1
			7,	6	5	4,	3	2	1
		8	7,	6	5	4,	3	2	1
	9	8	7,	6	5	4,	3	2	1
1,	9	8	7,	6	5	4,	3	2	1
2 1,	9	8	7,	6	5	4,	3	2	1

Students who learn to read large numbers are encouraged and motivated to learn more, because the task was simple and the results impressive.

Number Facts

In the work I do with students who learn differently, I have found that there are differences in the way people remember the basic number facts for addition, subtraction, multiplication and division. The following is a hierarchy of number fact recall.

1. Automatic Recall
2. Delay in Automatic Recall
3. Number Relationships
4. Number Tricks
5. Number Fact Guesses with Occasional Errors
6. Counting or Sequencing
7. Counting or Sequencing with Errors

1. Automatic Recall: The automatic recall of the number facts without hesitation is the mastery level. For example, when asked what 5 + 5 equals, most people automatically respond 10. This response is like an automatic motor response, such as a flinch when something surprises the person. Some individuals, especially those who have natural ability in math, have automatic recall of all the number facts. Others have automatic recall of only a few of these facts.

2. Delay in Automatic Recall: Some people know the number facts but they must think about some of them for a moment. They do not need number relationships or tricks to remember, just a moment or two to recall the number fact. This delay may come in the way the student processes the information. Some may have to visualize the two numbers as a problem, others may have to repeat the numbers to themselves.

3. Number Relationships: The next level of number fact knowledge is that of number relationships. With this technique the person combines numbers or uses patterns to determine the number facts. For example, when adding 8 + 6, the person automatically recalls 8 + 8 and subtracts two. Another example of number relationships is the patterns of Tic Tac Toe Math (described under multiplication).

4. Number Tricks: These mnemonic techniques provide the person with visual or auditory clues to remember some of the number facts. These techniques include music, rhymes, visualizations and language clues. For example 6 + 6 equals a dozen, a dozen equals 12.

5. Number Fact Guesses with Occasional Errors: Some individuals try to recall the number facts by guessing. These students often make errors and are usually unaware that their guesses are incorrect.

6. Counting or Sequencing: Many people who do not know the number facts count to calculate them. They may use their fingers, or count silently to themselves. Some make marks and count them.

7. Counting or Sequencing with Errors: Some students make errors when counting. One reason is that a person who has difficulty sequencing may skip numbers as they count or a person with a right/left discrimination problem may not remember whether to begin counting on or after the number. For example, 8 + 5 = 13, but if you start counting on the 8, the answer you arrive at is 12.

There is a natural progression from the bottom of this skill list to the top. Students who know the number facts with automatic recall usually possess natural ability in math, good quantitative categories, ample success with math and sufficient time on task. Students who have not reached the automatic recall level of the number facts usually know some of the number facts by automatic recall, such as 2 + 2 and 5 + 5, but often use lower level skills to recall or calculate the facts which they use less often.

Many students are not able to, of have not yet reached, automatic recall of the number facts. This happens for a number of reasons. For example anxiety, for whatever reason (performance anxiety or worry about every day problems), can result in a delay in recall. Students who avoid math do not use the number facts enough to remember them all. Students who have a right/left discrimination problem have difficulty remembering number facts and when counting, cannot remember if they start on the number or on the next number.

Addition

Individuals who count do not add; they sequence rather than group numbers. Not only is counting time consuming, it limits the individual's ability to learn higher math skills. The goal is automatic recall, but as noted in the previous section, this is not easily obtained or possible for many students, especially those with learning problems. However, there is more to addition than learning the number facts. Adding is the grouping of numbers (single digits, numbers with multiple digits and columns of numbers) and includes operational concepts like carrying. Similar to the number facts, there are a number of reasons why students may have difficulty with one or many of the concepts or operations involved in adding. The following techniques will be useful in teaching addition facts, operations and concepts especially to those with learning problems. Students who have learning problems, or who are starting from the bottom of the number fact recall hierarchy, should try many techniques at higher levels of proficiency until automatic recall is possible.

Numbers in Numbers: One of the first things that students need to do is to see numbers in numbers. For example, they need to see that 5 is made up of 5 ones, 2 two's and a one, a three and a two, etc. Students should be encouraged to explore the number combinations. For some students, this exploration will lead to mastery of the number facts with automatic recall. Other students will learn number relationships and reduce the need for counting.

						1
	4	3	2	2	3	1
5 =	1	2	2	1	1	1
			1	1	1	1
				1		1

Patterns: I find that it is helpful for students to see all of the single digit number facts. When one looks at these number facts, the patterns are uncanny. Some students see them immediately; others have to have the patterns pointed out to them. The patterns can be used by many students to remember number facts. On the following pages, notice the following patterns in the addition facts.

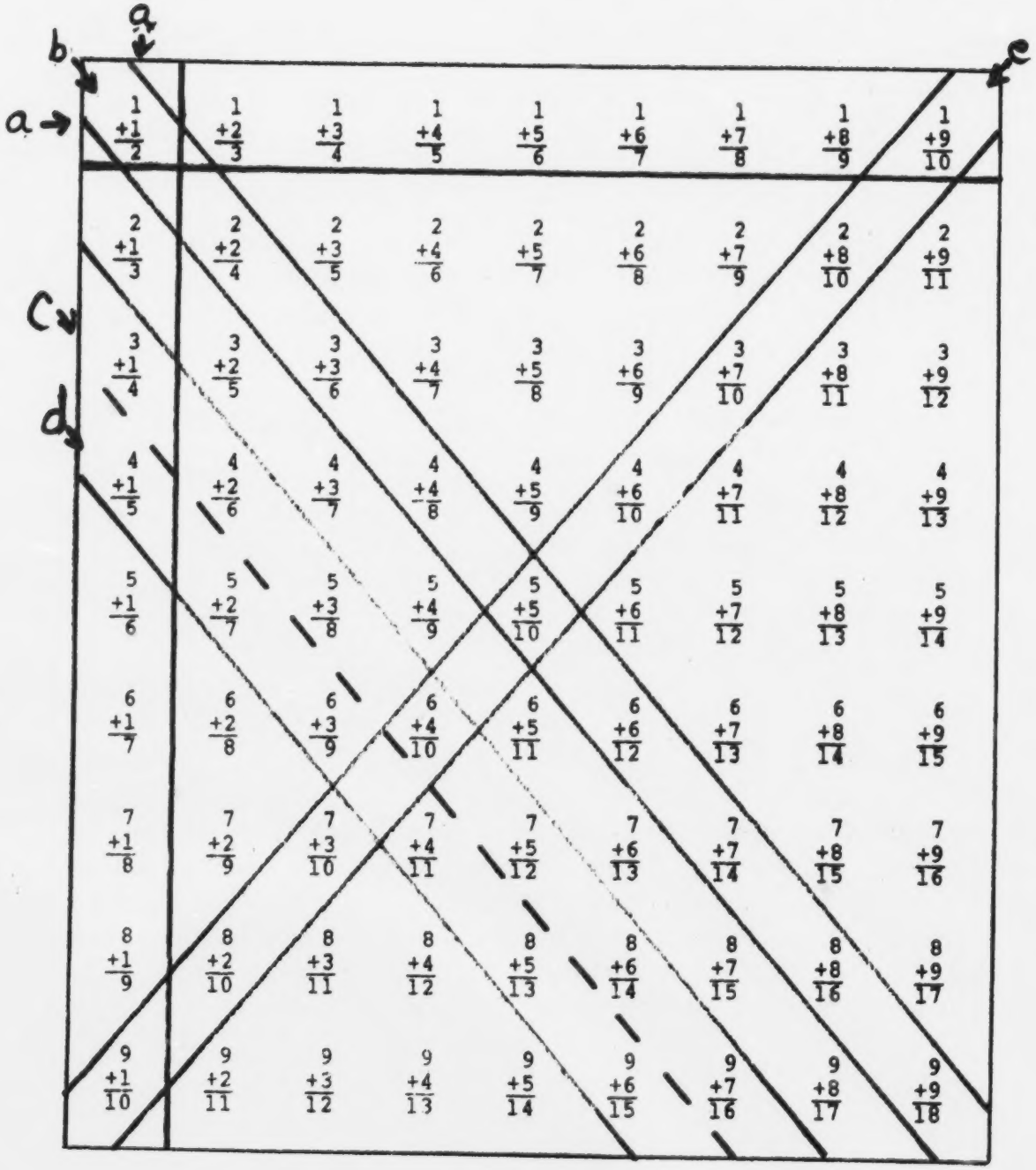
1. Horizontally and vertically the number facts increase by one (a).

2. Looking at the patterns on the diagonal, (b) students can see that adding two of the same number, 5 + 5, always equals an even number. The concept of doubling the number should be introduced if it is not already understood.

3. Looking at the diagonal going the same direction, the student can see that the sum of number facts increase by two, one diagonal column being even (c) and the other being odd (d).

4. Looking at the diagonal going in the opposite direction, the student can see that the sum of number facts are the same for each fact along this column (e).

The following pages can be used to help students identify the number facts they know and do not know. Once identified, the facts the students do not know can be learned through a variety of techniques. (Note: the assessment of the number facts should take place over a number of days. If all the number facts are tested at one sitting, most students will be able to use their knowledge of some facts to remember others. Whereas, when the assessment of number facts takes place over a couple of days, the student's usual way of recalling the facts will be evident.)


a
b
a
c
C
d
1 +1 2
1 +2 3
1 +3 4
1 +4 5
1 +5 6
1 +6 7
1 +7 8
1 +8 9
1 +9 10
2 +1 3
2 +2 4
2 +3 5
2 +4 6
2 +5 7
2 +6 8
2 +7 9
2 +8 10
2 +9 11
3 +1 4
3 +2 5
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5 +6 11
5 +7 12
5 +8 13
5 +9 14
6 +1 7
6 +2 8
6 +3 9
6 +4 10
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6 +9 15
7 +1 8
7 +2 9
7 +3 10
7 +4 11
7 +5 12
7 +6 13
7 +7 14
7 +8 15
7 +9 16
8 +1 9
8 +2 10
8 +3 11
8 +4 12
8 +5 13
8 +6 14
8 +7 15
8 +8 16
8 +9 17
9 +1 10
9 +2 11
9 +3 12
9 +4 13
9 +5 14
9 +6 15
9 +7 16
9 +8 17
9 +9 18

13

Specific Number Fact Techniques: The number facts that add to ten are important for the students to know by automatic recall. The following mnemonic, or trick will help the students who have difficulty remembering these facts. By using this method they will always get the facts correct and often over time this develops into automatic recall. If not, the student can continue to use the "trick".

The person writes as they say, one, two, three, four, five and five are ten. Then (under the four) six, seven, eight and nine.

1	2	3	4	5
9	8	7	6	5
		10		

Here are three methods for recalling number facts involving nines: The first uses ten as a base. When adding 9 to any number, one increases the 9 to a 10; state the other number as a teen and subtract one. For example, 9 + 6. The 9 become a ten, 10 + 6 = 16 minus one is 15.

Another way is to use a number key (see Number Keys below). 9 + 7 is the same as 9 + 9 minus 2.

A shorter way and one which many students with learning problems find easy to remember, is 9 + any number = one less than that number "teened" So 9 + 7 = 6 "teened" or 16. 9 + 3 = 2 "teened" or 12.

Number Keys: The numbers that add to ten (9 + 1, etc.) and the numbers doubled (2 + 2, etc.) can be used in a system that I call number keys. In this system, the student learns the keys (numbers that add to ten and doubles) and learns to add or subtract one and then two from these keys (using number relationships) to recall the number facts. The following is an example of a number keys exercise page.

5	5	5	6	6	6
+5	+4	+6	+6	+5	+7

7	7	7	8	8	8
+7	+6	+8	+8	+7	+9

Dice: A set of at least six dice can be an effective tool for teaching some of the addition number facts. The student rolls the dice and groups the numbers into groups of ten dots. The remaining number of dots is added to the 10, 20 or 30 from the dice that have been grouped. For example, if the following combination was rolled, there would be two groups of ten and one remaining die showing three dots.

The number of rolls of the dice vary depending on how quickly the student learns the number facts that add to ten and how long the exercise sustains the student's attention. In addition to helping the students learn the number facts that add to ten, this exercise helps develop organizational skills.

Larger Numbers: Many students with weak math skills do not even realize that large numbers can be added without writing them out first. Students should be shown that they can learn to add large numbers mentally. For example, 20 + 5 and 100 + 75, etc.

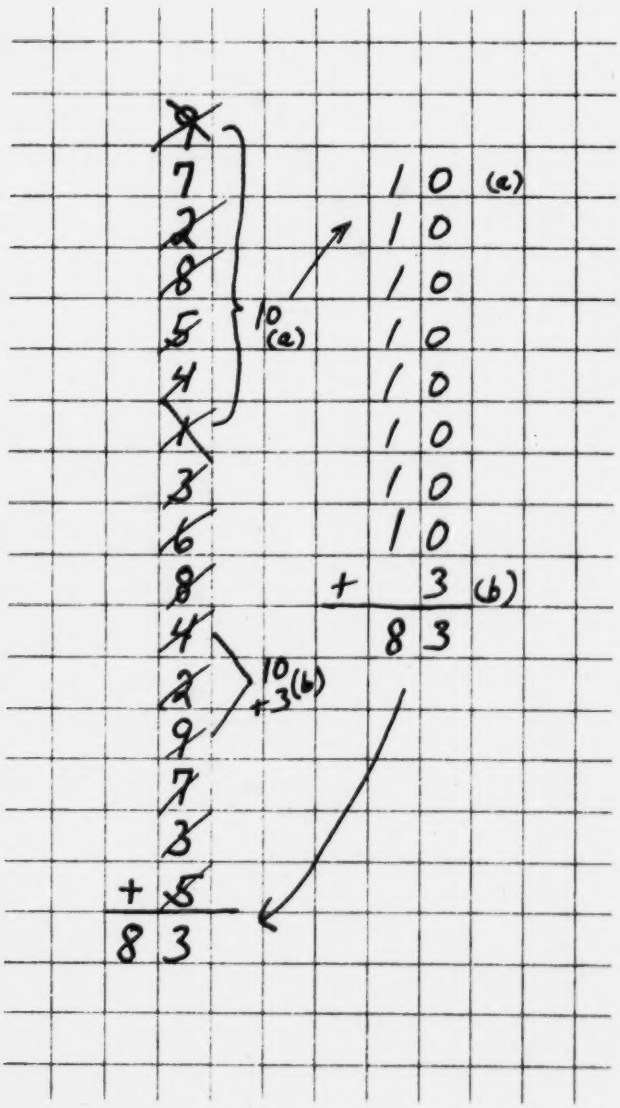
Language cues can also be used as mnemonic clues to remember some of the number facts.

1 + 1 2 eyes 2 + 2 4 limbs 5 + 5 10 fingers
6 + 6 12 in a dozen 7 + 7 14 a common football score
8 + 8 16 - an eight reminds you of a race track and 16 is associated with the age one can obtain a driver's license in many states.

Adding Columns

There are a number of ways to add columns of numbers. Students without difficulties in math can remember the numbers as they add each number in a column. Students who are weak in math often cannot hold these numbers in their heads. They forget the numbers as they add or it triggers to another number. For example, when adding 9 + 8 + 5 + 4. The student who has difficulty remembering numbers might follow this sequence: 9 + 8 = 17 + 5 = 22; 23+ 4 =27 I call the jump from 22 to 23 a trigger. This explains how a student can make an error even though the person can add. (A trigger is a jump in the thought process. The phenomenon of triggering is often observed in individuals who have a racing mind, another way to describe attention deficit disorder.) The following are a number of alternative ways to add columns which help students with various math weaknesses.

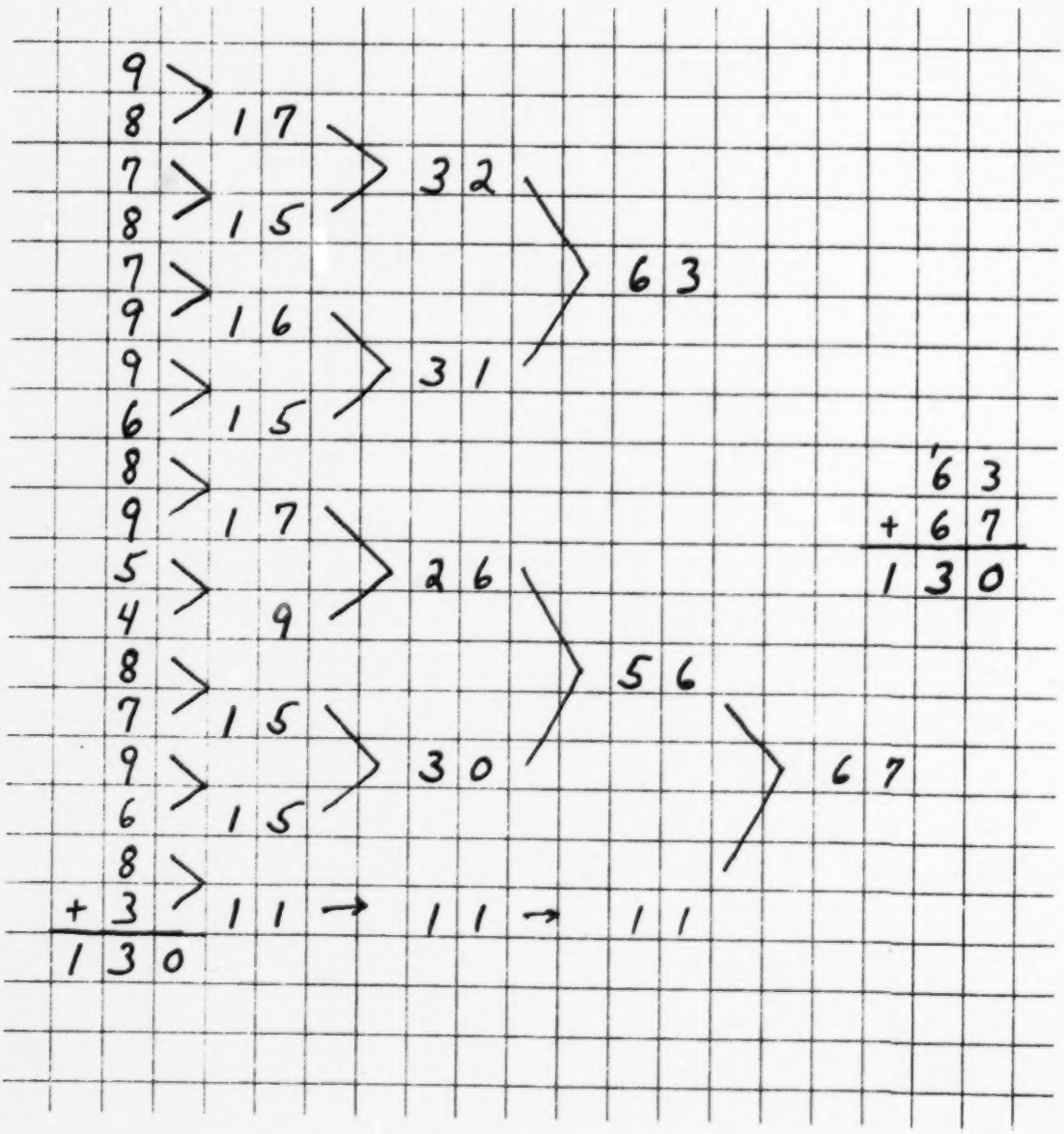
Strike Ten: This method is an easy way to add a column of mixed numbers. The student strikes out any two or more numbers that add to ten. Each ten is written next to the column or on a separate piece of paper. The tens are then added along with any number which remains. Here is an example.



Students check their answer by striking the number again making an X. (a)

Note that when adding 9 + 4, the student takes one from the 4 to make a 10 with 9. (b)

Pairing: Some columns of numbers do not have a gook mix to easily produce combinations of ten. These columns can be easily added by pairing. Adding columns this way can also be easily checked. This is important for students who are weak in math. Because all the calculations are visible, the student can check each one quickly.



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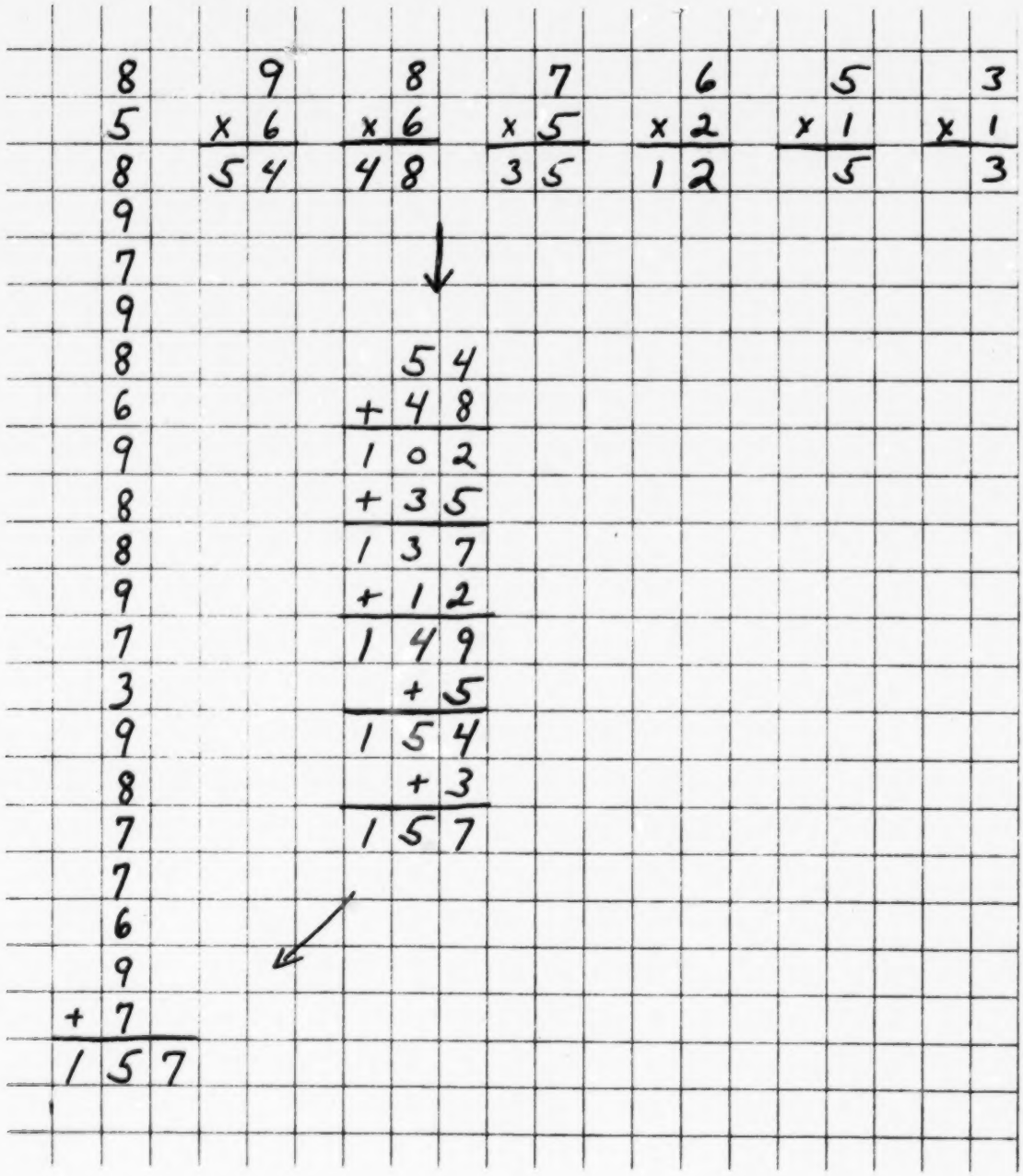
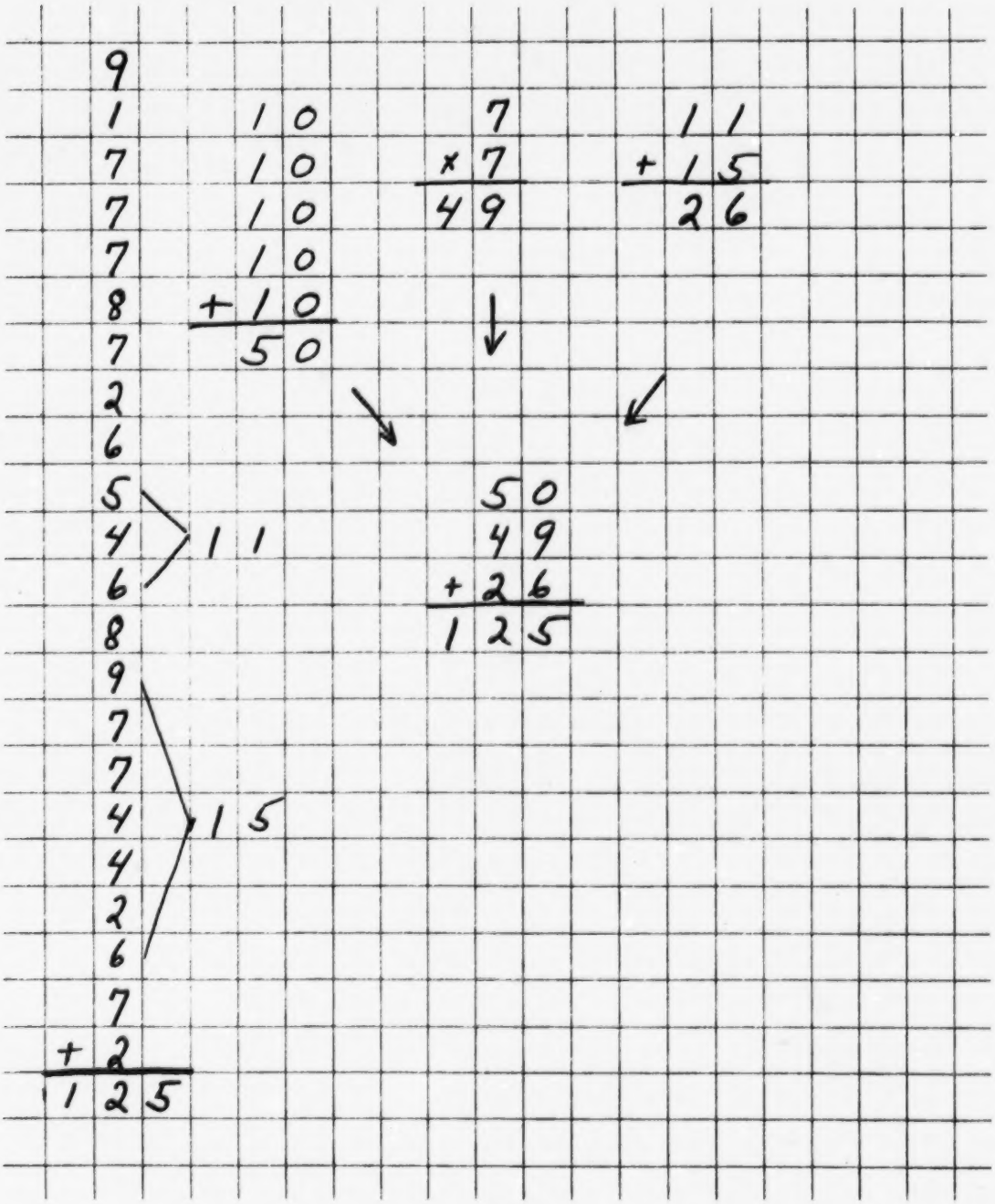
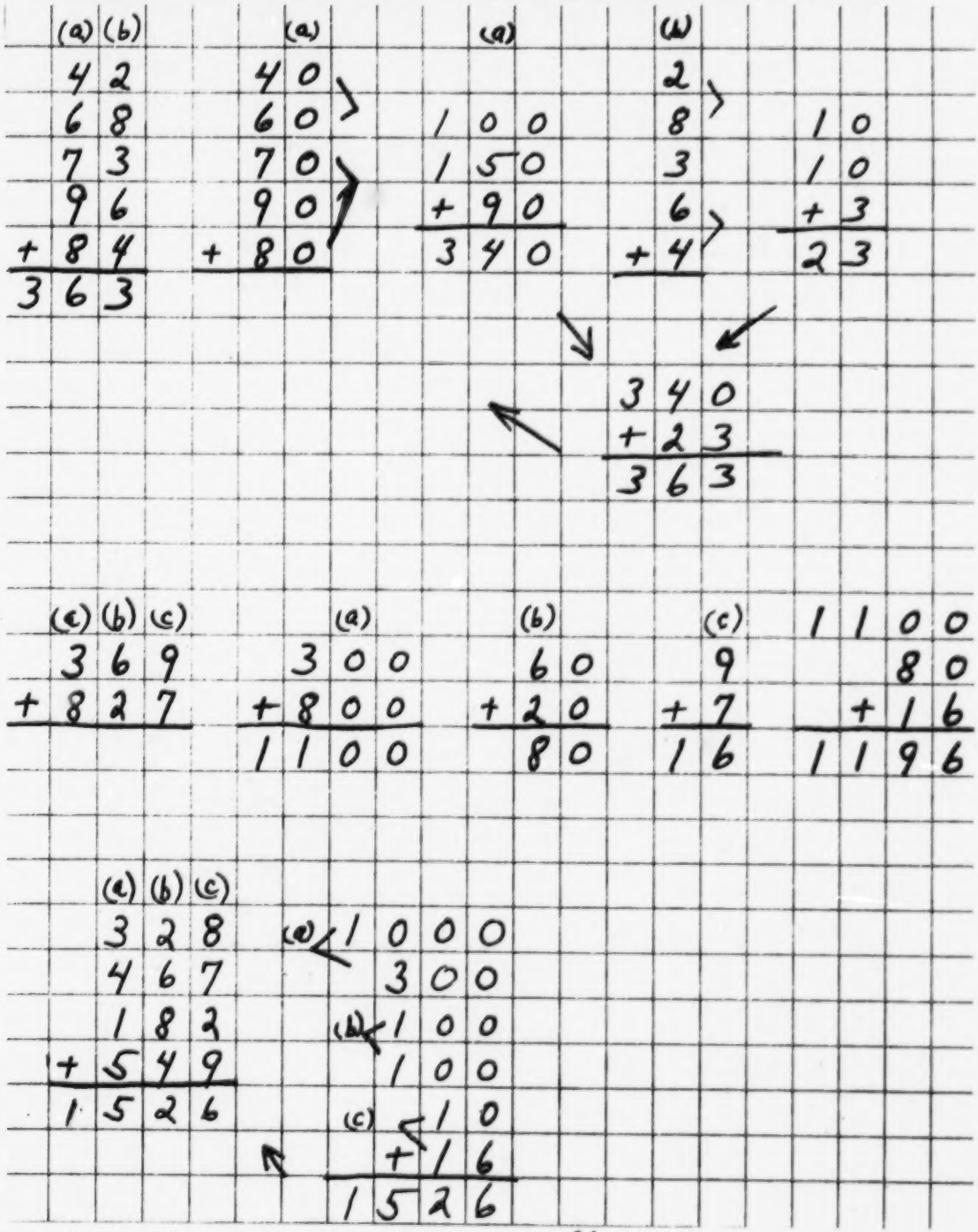
Multiplying: Still, another method for adding columns is to count how many of each number there are and multiply that number by the number of times it appears. The products can then be added, using the previous methods of strike ten or pairing. Study this example.



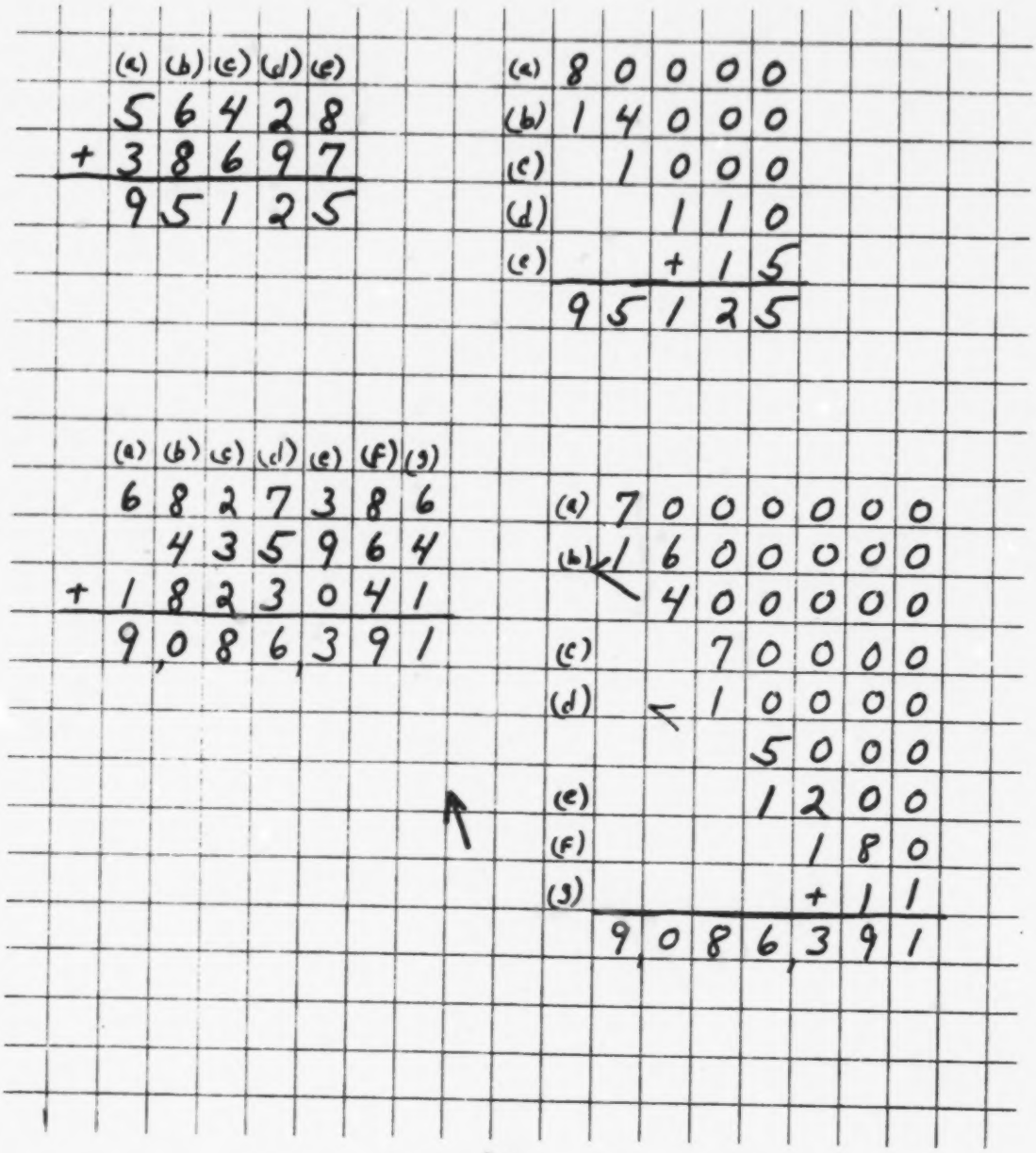
Mixed Techniques: There is nothing to prevent the student from mixing the techniques as the following examples illustrates.



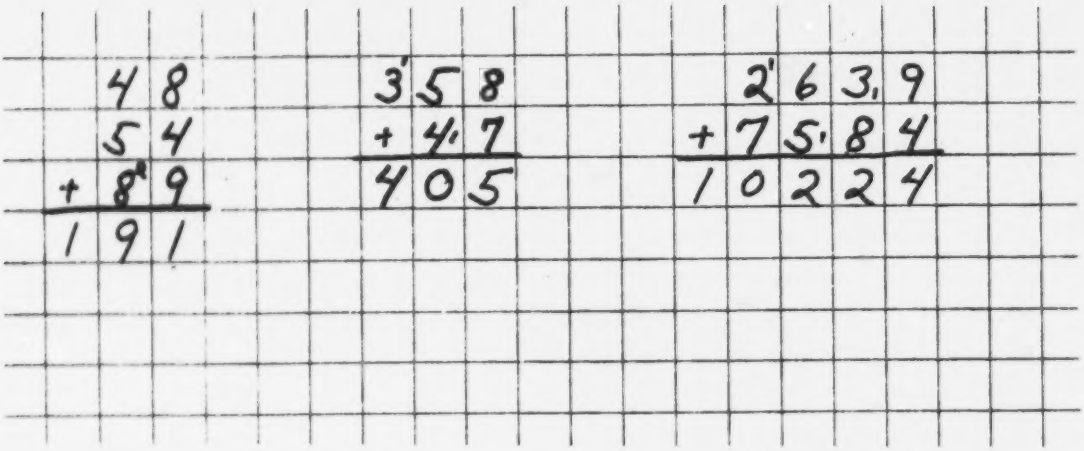
Rewriting with zeros: Columns with double-digit numbers can be added by rewriting the numbers to obtain many zeros. This helps the student who has difficulty with carrying and those who find it difficult to hold numbers mentally. The following examples illustrate this technique.



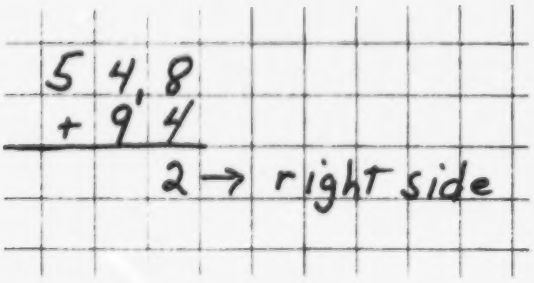
Adding Larger Numbers: The same principle of breaking the large numbers into parts to obtain more zeros can be used as in the following demonstration. Some students need to be taught to count the number of places to the right of the column one is adding and convert these to zeros.

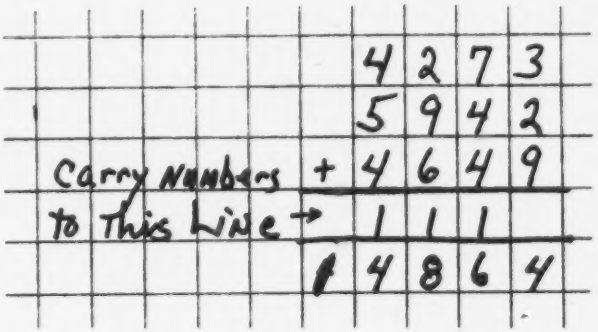


Problems with Carrying: Many students with moderate to severe learning problems have difficulty with the concept of carrying. There are a number of techniques which may help these students. The first is to teach them to carry to a friendly number rather than to automatically carry to the top of the problem. A friendly number is one which will combine to make a ten or another number which the student can easily add.



For those with a right/left discrimination problem, mnemonics can be developed to help the student remember which number to carry. For example, a student may learn to feel his or her right side and associate that with the number which is written under the line, while the number to be carried is associated with a feel for the left side of the body. Another technique for carrying is to leave a space so thta the numbers carried are close to the number which are not carried.





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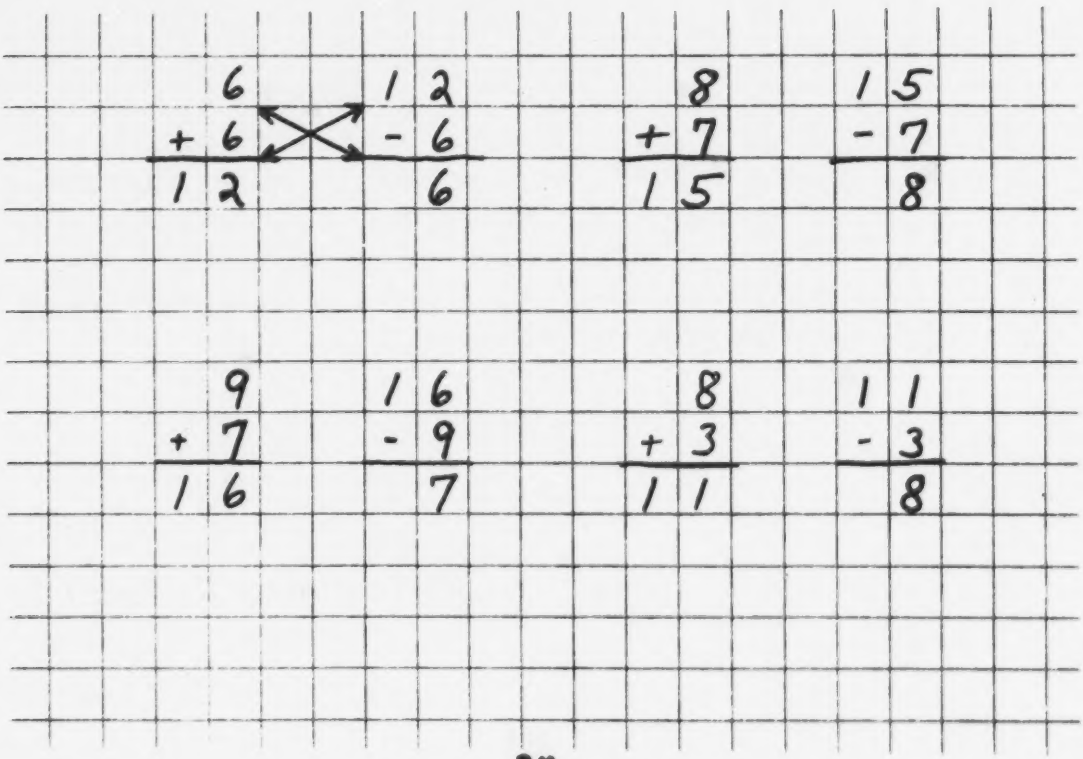
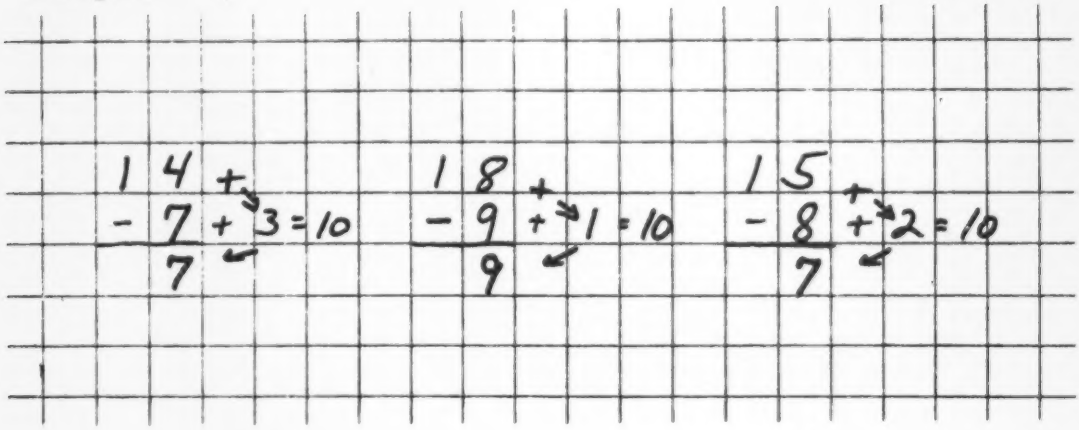
Subtraction

Students who have difficulty with addition have even more difficulty with subtraction. Many students, who know some of the number facts using number relationships, count when subtracting. Some students who count still come up with the wrong answer because of various processing problems. The following are some techniques to help students with subtraction. As with the other number facts, automatic recall is the goal, but any proficiency at a higher level is preferable to counting.

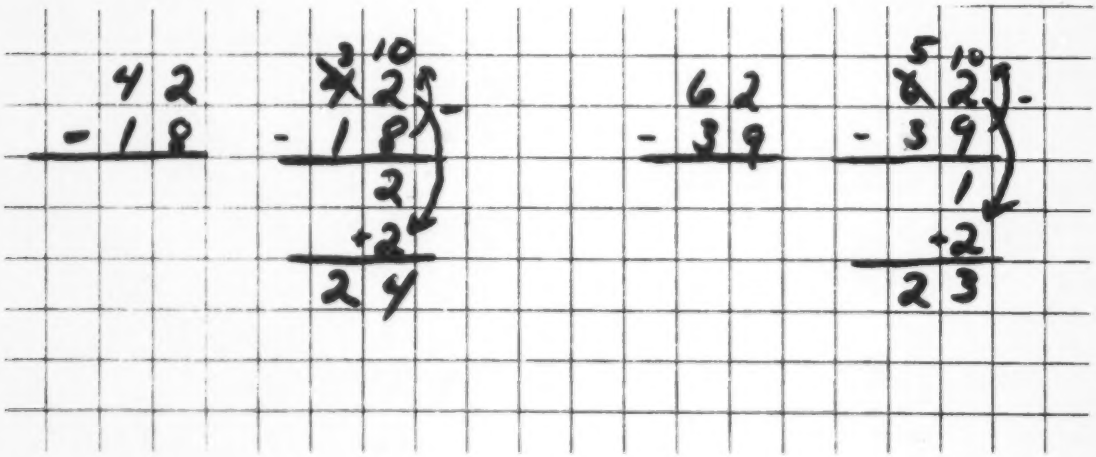
Reverse Addition: Students should be shown that subtraction is the reverse of addition. This simple concept is not obvious to all, and it should not be assumed that students automatically see that the addition facts are involved. The following examples demonstrate that there are similar patterns in the subtraction facts and that these patterns, or facts, are indeed the opposite of the addition facts.



Subtracting by adding: This technique uses a trick to allow the student to subtract by adding. For subtracting a single digit from a teen number, the student places, and eventually visualizes, the number which would add to ten next to the bottom number. This number is then added diagonally to the top number in the one's column. Study the following examples.



Subtracting with a borrowed ten: Some students, especially those who have difficulty with visual processing, find borrowing difficult. A technique which I have seen students use effectively is shown below. This techniques allow them to see the numbers better and remember what they have done.



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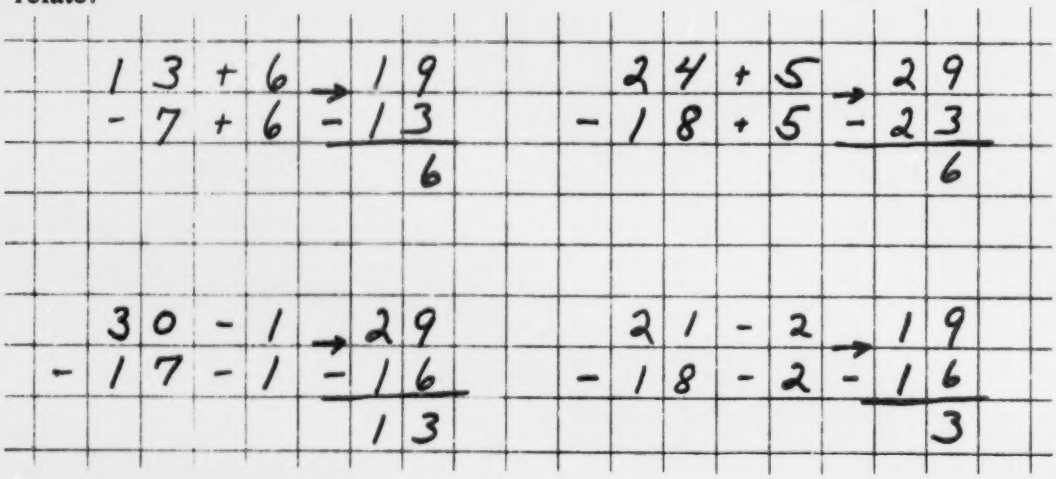
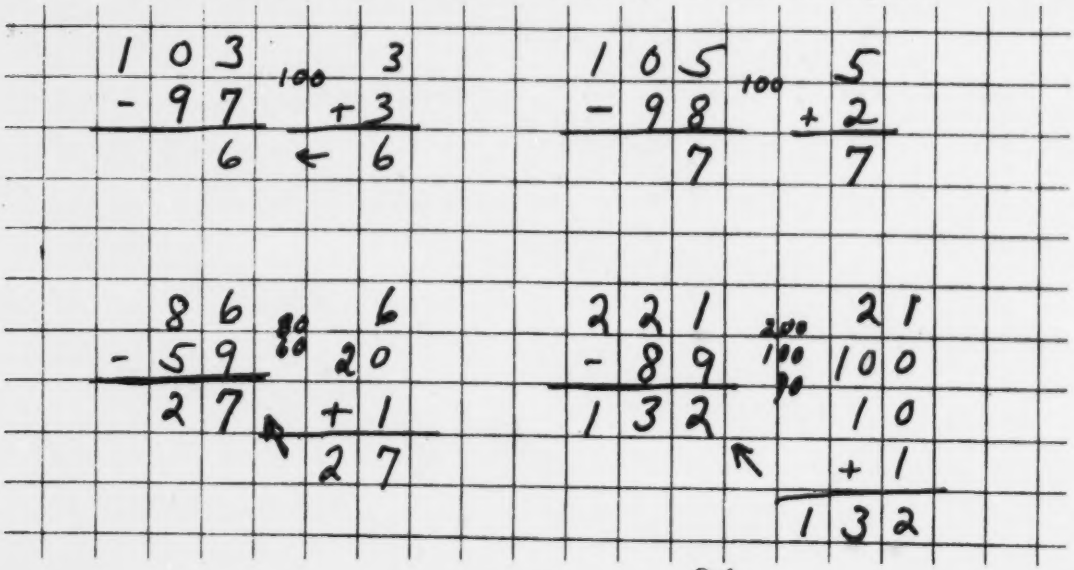
Adding or subtracting to eliminate borrowing: This technique allows the student to change the numbers so that there is no need to borrow from the ten's column. In the following examples, the numbers are rewritten to allow the student to subtract. This is not a common technique; but, it is worth demonstrating to students because for some it will make sense and for others it helps them to understand how numbers relate.



Subtracting without borrowing: This technique works very well with some numbers and as the student continues to use it, other number combinations become easier. In the following example the student writes, and eventually visualizes, the number which lies on the number line between the two numbers. Here 100 lies between these numbers. The student then sees that 103 is three away from 100 and that 97 is three numbers away from 100. 3 + 3 are six. Study the other examples.



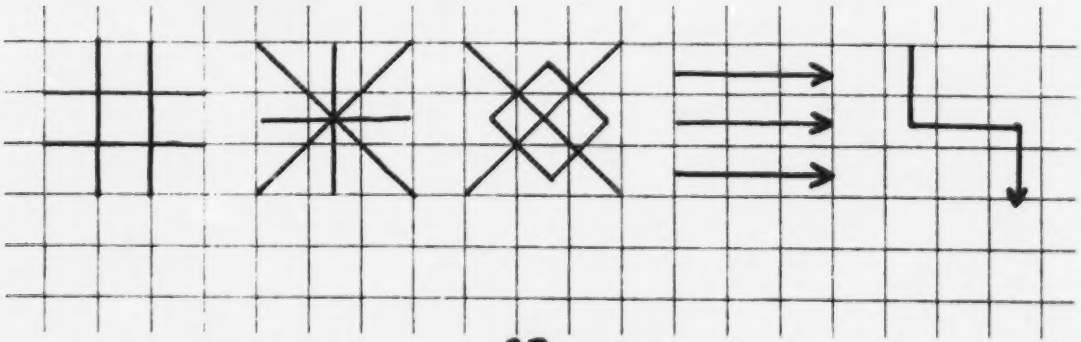
Multiplication

The number facts of multiplication are more difficult for some students because the patterns are less obvious and the numbers are larger. However, I have found there are students who count the addition facts but know the multiplication facts. This phenomenon occurs because the students are instructed to memorize the multiplication facts, but are allowed to count the addition facts. However, it is more common for students with math problems to have difficulty understanding the progression of multiplication number facts because they have difficulty seeing number relationships and knowing mnemonic tricks. Since counting is much more time consuming for multiplication, the most common responses are number fact guesses or avoidance.

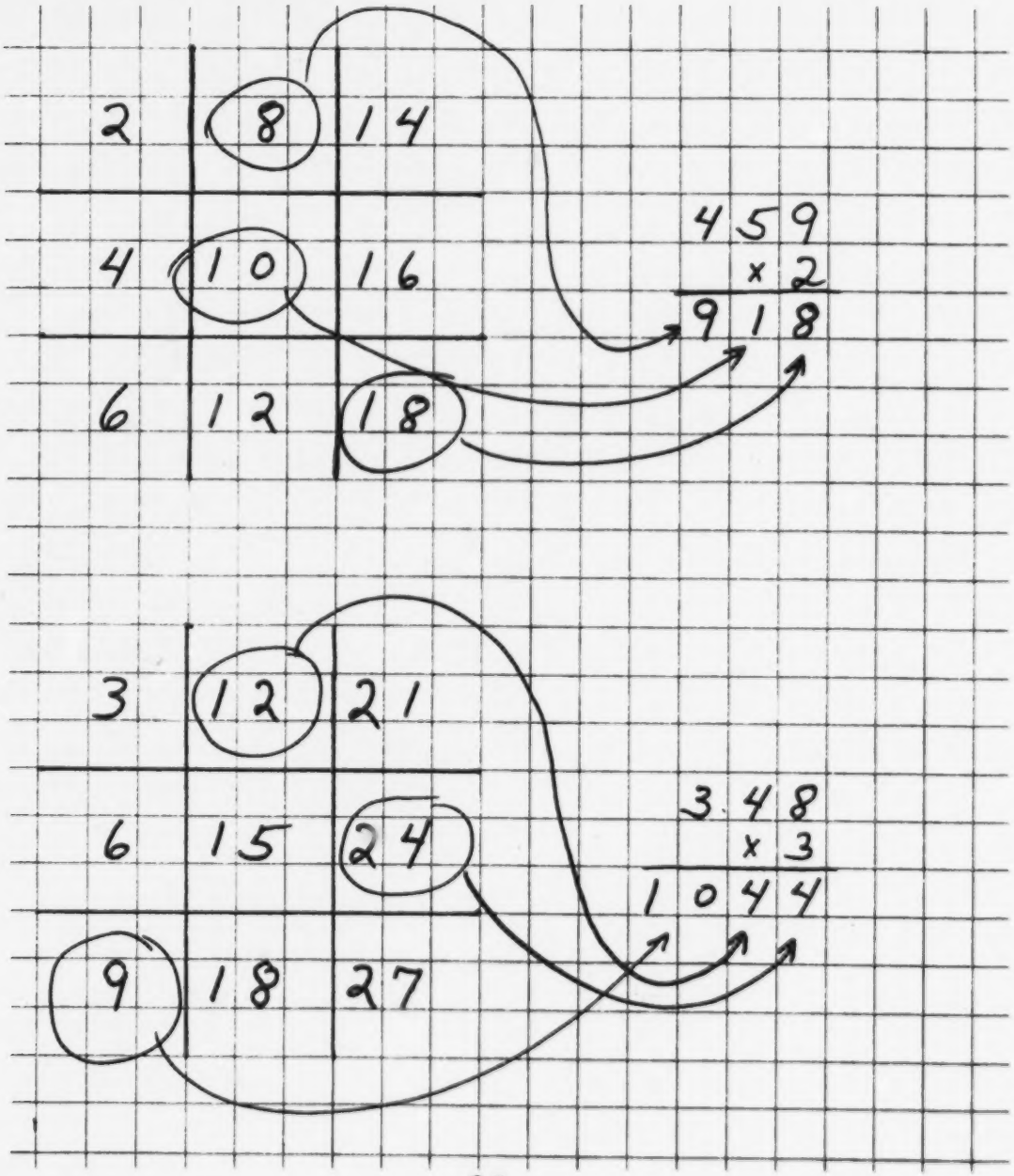
Students should be encouraged to learn the multiplication number facts with auto recall, but there are a number of learning problems which make this task extremely difficult. If a child has not been able to learn the times tables during the year after his or her chronological peers, then alternative methods should be tried.

One of the most effective number relationship techniques for those students who have not been able to learn the multiplication number facts by automatic recall is Tic Tac Toe Math (Cooper, 1988). This method uses visual patterns to allow the person to calculate the multiplication tables one through infinity. Tic Tac Toe Math uses the nine positions of a grid to make the tables. This system differs from simply referring to a chart of the times tables in that the student does not have to have a times tables chart. Instead, the grids are made when they are needed for solving problems. In addition, using the grids enables the students to produce the times tables for every number through infinity.

Some students, especially those who are good with visual patterns learn the patterns of Tic Tac Toe easily. The following patterns are used in Tic Tac Toe Math.



Once the student learns how to make the grids, multiplication problems are easily solved. The grids are like a multiplication chart; but, the student creates the grids instead of relaying on a grid prepared by someone else. This makes the student more independent and increases self-esteem. For more information about Tic Tac Toe Math contact Learning disAbilities Resources, 1-800-869-8336.



Singing the Times Tables: Some students find that musical melodies or rhythms are good auditory mnemonics which enable some students to remember the multiplication facts. A note of caution is that the number of people who can learn the number facts using music is probably smaller than by other methods. If you are a person who uses music or other auditory clues, do not expect others to be able to use this method. Try it; but if it does not seem to be an easy task, try something else.

Mnemonic Clues: Mnemonic clues, or memory aids, can be developed for some of the multiplication facts. Not every number fact is difficult to remember; some of them just seem to be more difficult than others. Here are a few mnemonics for remembering some of the multiplication facts which many students have difficulty recalling.

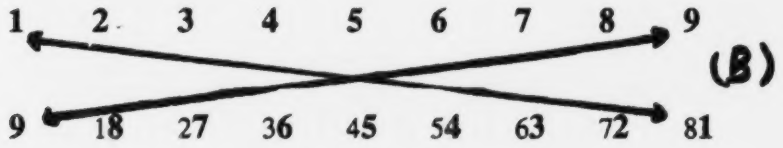
666	6 x 6 = 36
5678	8 x 7 or 7 x 8 = 56
Football	7 x 7 = 49 the San Francisco 49er's

The two digits which are the products of the nine times tables added together result in 9 for example 9 x 2 = 18 (1 + 8 = 9).
With a little creativity students can develop mnemonics for the number facts that are difficult for them to remember.

Counting Exercises: The following exercises can be both motivating to students, because they are able to do things which they thought were beyond them, and the rhythm of the sequence number can help some students learn the multiplication facts. The student uses the number in a repeating pattern to count by 2's, 4's, 6's, or 8's.

2 4 6 8 0	The following example shows how a person would use the patterns.
4 8 2 6 0	
6 2 8 4 0	
8 6 4 2 0	8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120

Patterns: Some students find that patterns are useful as aids in learning the multiplication facts. Pointing out the odd and even patterns (Example A), or that the numbers in the one's column are the reverse of another table (Example B), can be used by some students to remember the facts.

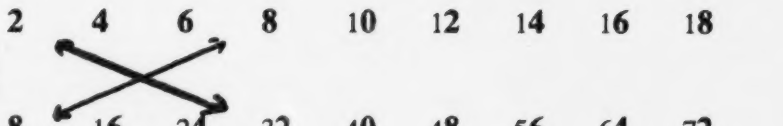


(B)

odd (A)

3	6	9	12	15	18	21	24	27
7	14	21	28	35	42	49	56	63

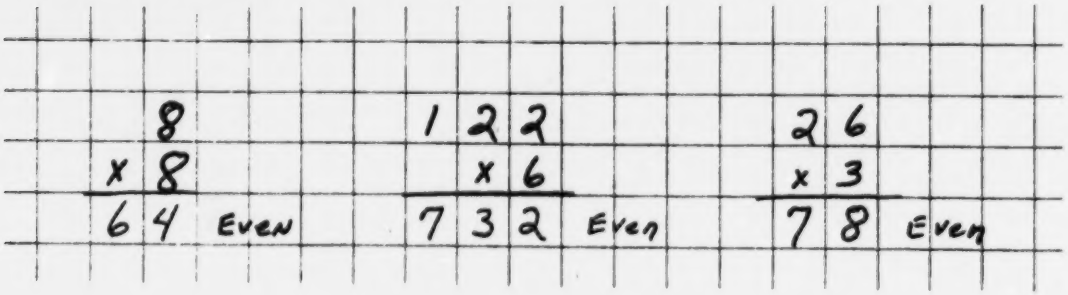
(B)



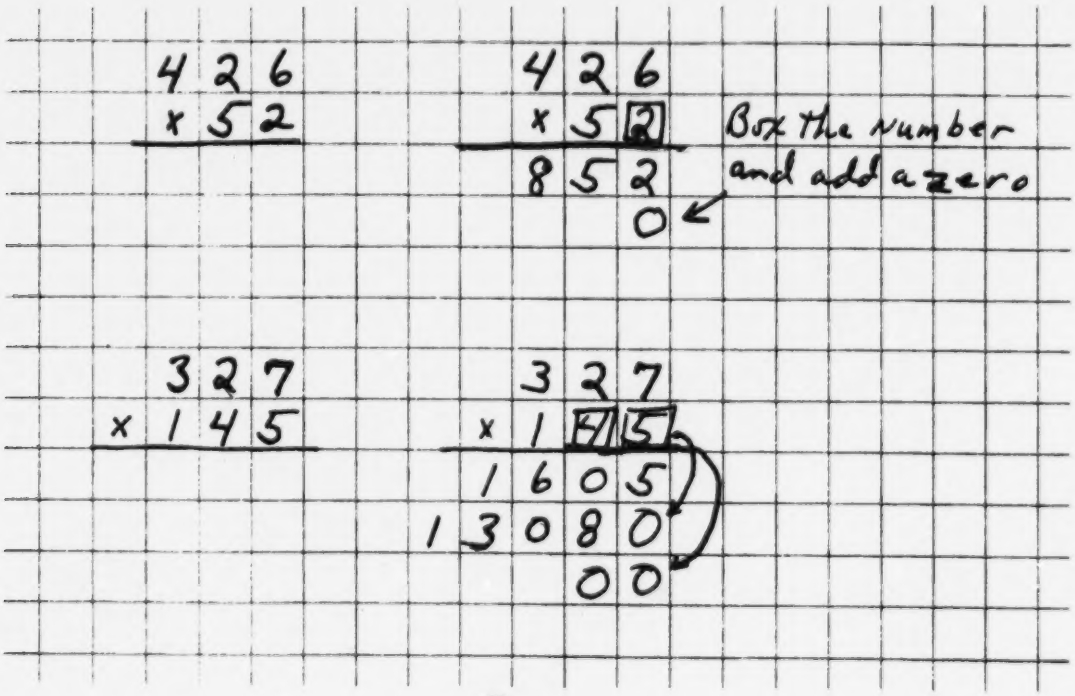
even (A)

4	8	12	16	20	24	28	32	36
6	12	18	24	340	36	42	48	54

As with the addition facts, the odd and even patterns can help students remember the multiplication facts. Tic Tac Toe Math uses different visual patterns based on odd and even numbers. Another way to use odd and even is for the student to be aware that the product will always be even when multiplying any number by an even number.



Some students have difficulty understanding or remembering to use zeros for the place holder when multiplying with multiple digits. One technique which will help is to routinely place a square around the number or an X over it and place corresponding zeros as place holders (see examples).



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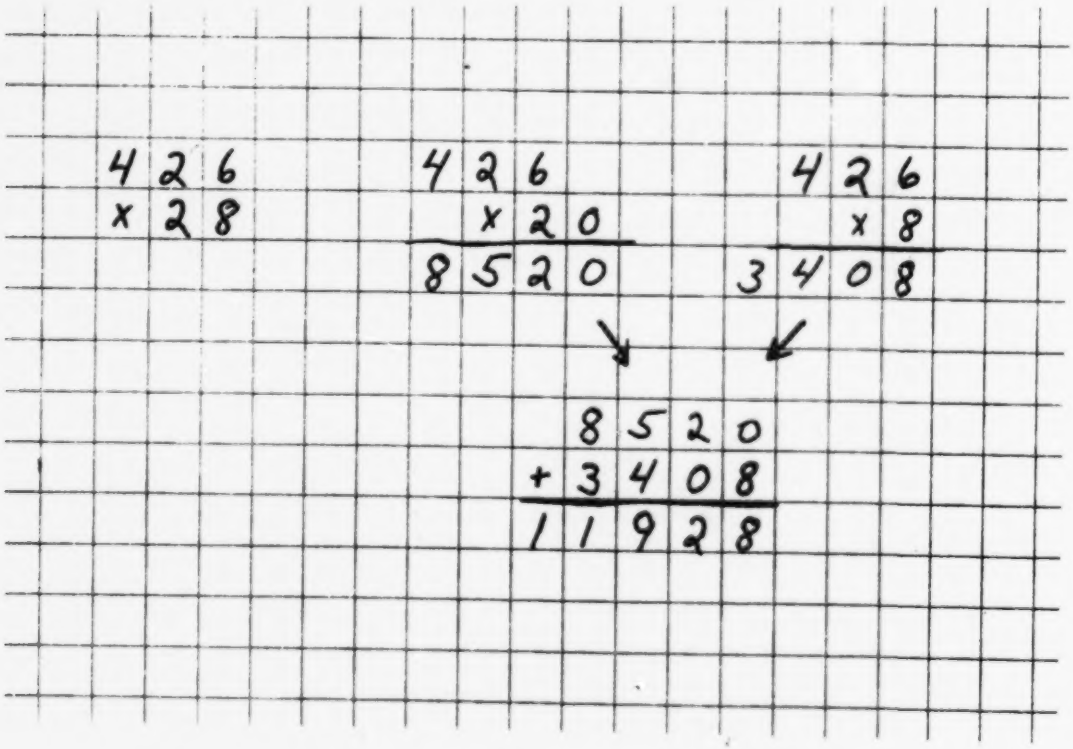
Breaking numbers apart: One way to help students understand the process of multiplying with 2 or more digits is to demonstrate that the number can be broken apart.

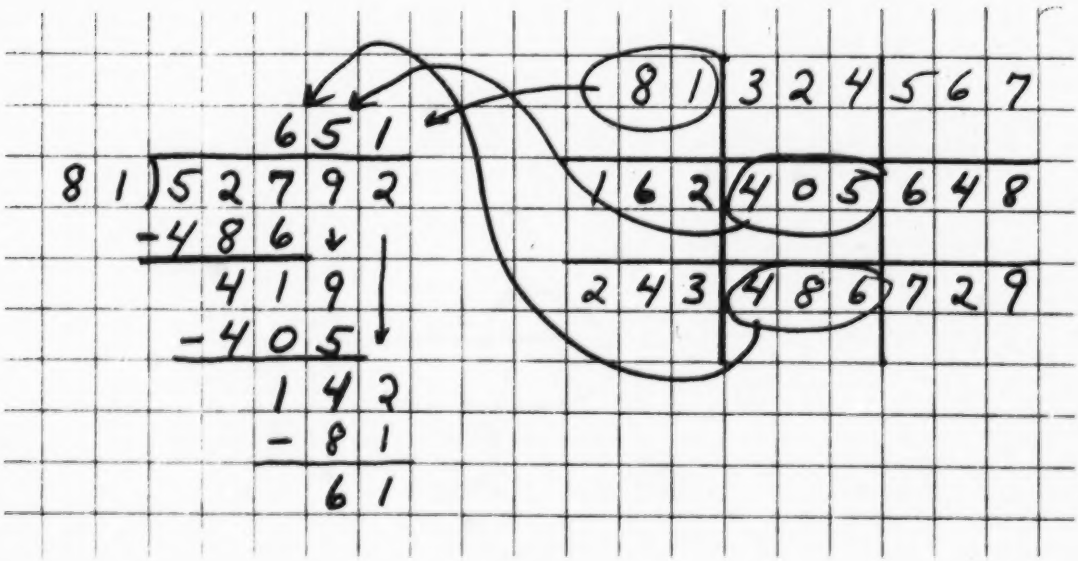


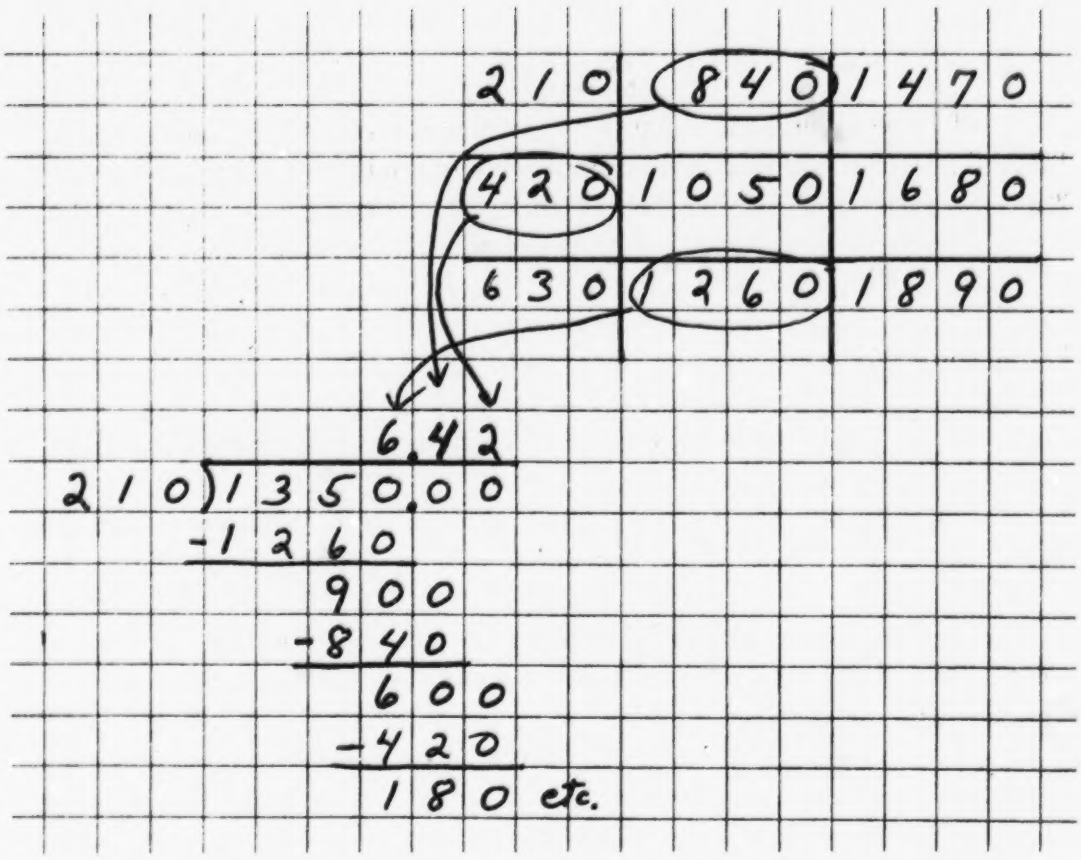
Division

Obviously, the division facts are the reverse of the multiplication facts; therefore, students who do not know the multiplication facts are not able to master the division facts. While some students use counting to do multiplication, these students usually are not able to reverse the procedure. Therefore, they cannot do division. These students usually resort to a multiplication chart which is a number trick in the number fact hierarchy. However, this does not help them with division larger that 12. Using graph paper may be necessary more for long division that for other math operations. This is true because of the way the numbers move down and across the page as the problem progresses.

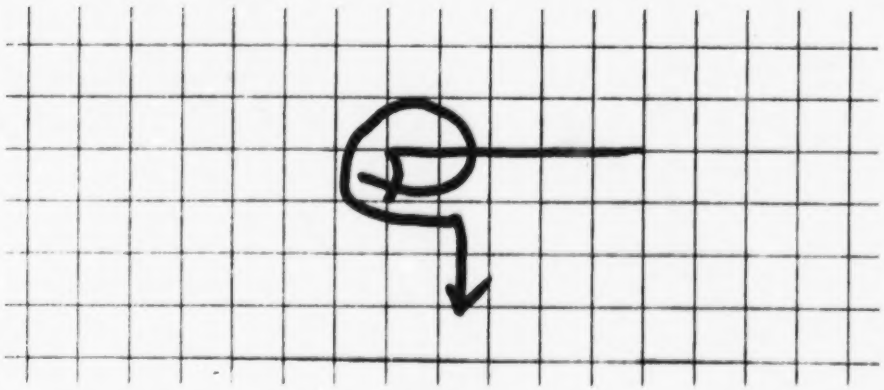
Tic Tac Toe Math enables many students, who otherwise could not learn long division, to learn and do it easily. This method allows the students, especially those with learning problems, to eliminate many of the difficulties they have with division and especially long division. It reduces the number of steps involved in dividing and eliminates estimating, multiplying and carrying.

The following examples demonstrate how a student would complete a double digit long division problem. Students learn to make the grids for any number through infinity. Then they simply match the numbers in the problem with those in the grids.





Visual or auditory mnemonic clues can help students remember the sequence of operations when doing division. The following visual pattern will help some students to remember the sequence for doing long division. Others will find the audity clue of E, M, S, Bd, R (Estimate, Multiply, Subtract, Bring down, Repeat) to be easier to remember.



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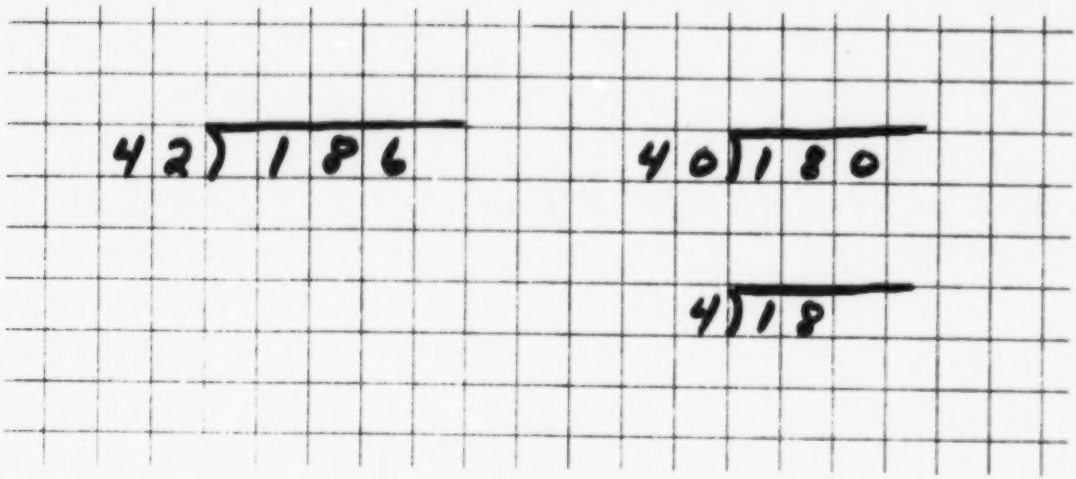
Estimating is difficult for students with weak math skills. One way to help the student improve their ability to estimate is to have them write out the estimate and demonstrate in the following example.

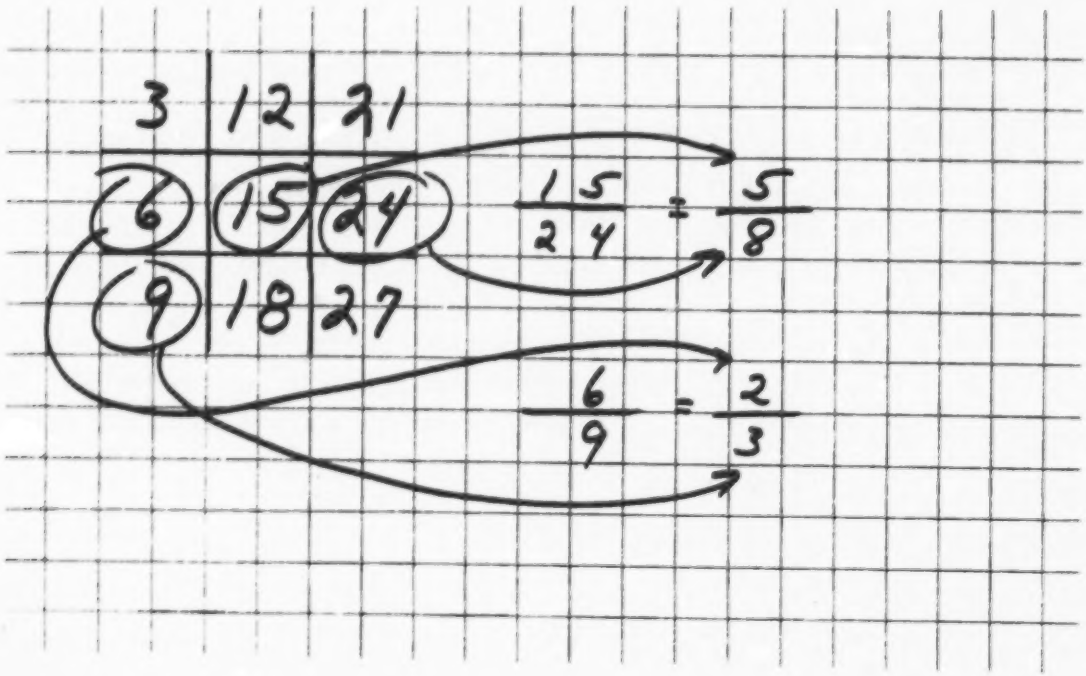


Many math exercises progress too quickly to more difficult problems for students with learning problems. In order to help students to master procedures and the sequences of operation, math exercises should be customized. This can be accomplished by adding more simple problems to each exercise and reviewing operations before each exercise. Another way to enable a student to remember the sequences of long division is to have the student describe the procedure without thinking about the number facts involved. By dividing the learning process in this way, the students is able to focus on the sequence of the operation until it is mastered.

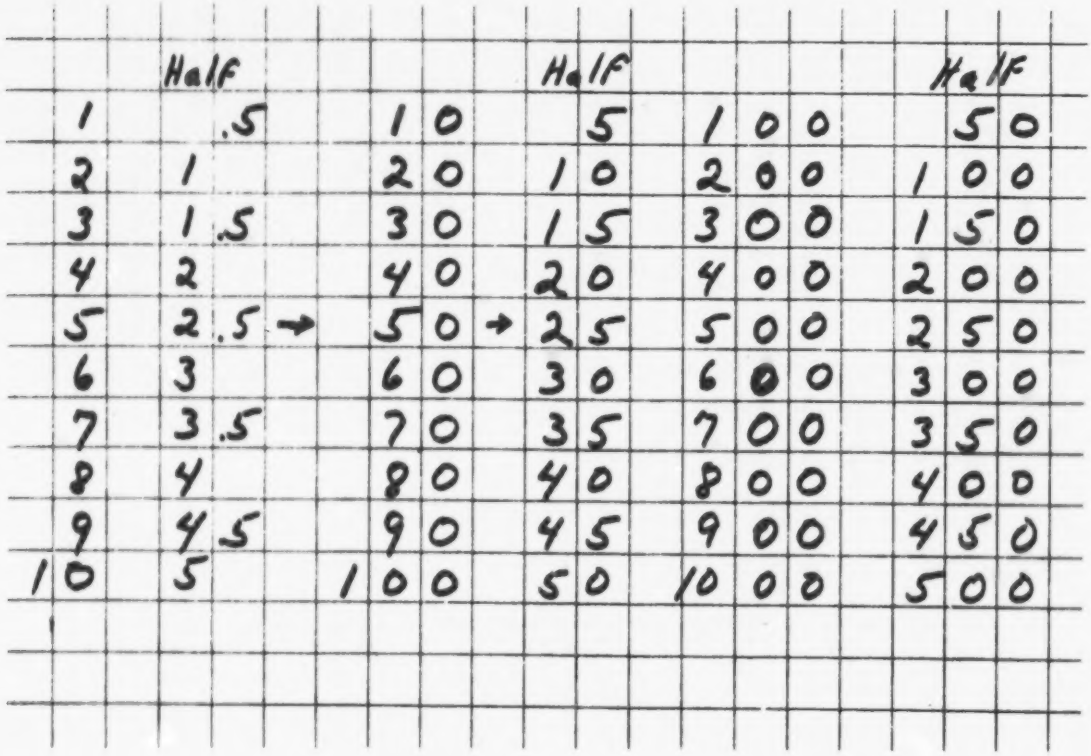
Fractions

Students who do not know the times tables cannot master fractions. Although these students may be able to learn to complete operations using simple fractions, they are not able to reduce fraction to the lowest terms or find common denominators. For this reason, students either need to learn the times tables or a system like Tic Tac Toe Math.

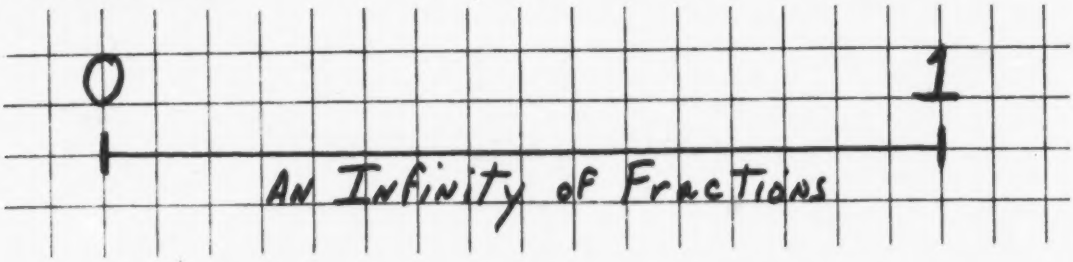
With Tic Tac Toe Math the student makes a set of grids and then can use them as other students would use the times tables in their mind's eye. The only difference is that the students who use Tic Tac Toe Math do not know the times table with automatic recall but rather use number relationships. Once the grids are made the student can easily work with fractions. Note the following example.



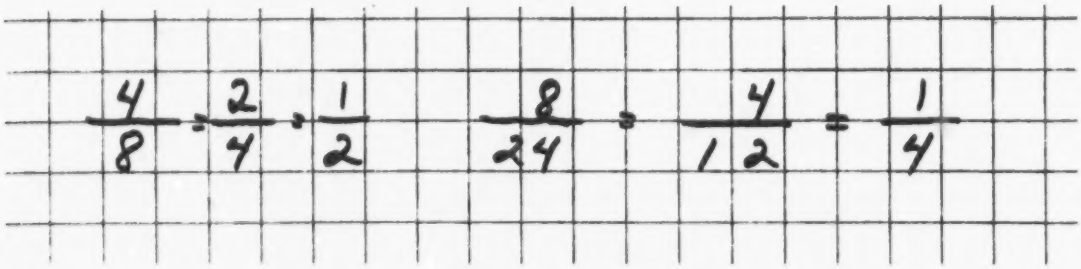
Introducing the concept of halves in the following way helps many students to understand fractions. The student is shown the numbers one to ten and then asked for the halves of the even numbers. Then they can be shown that the halves of the odd numbers are half way between the halves of the even numbers. Next, the students are shown how this same pattern exists in larger numbers.



I have found that many students do not understand what a fraction actually means. To these students the word fraction means only the numbers written with a line between them. They do not understand that mathematically a fraction means part of a whole. Conversely, when using language, these same students do understand that a fraction is part of a whole. They have no problem explaining that two halves make a whole. One way to help students to understand that fractions are parts of a whole and to make the infinite number of fractions seem more manageable is to explain that since fractions are parts of a whole , then all fractions exist between 0 and 1.



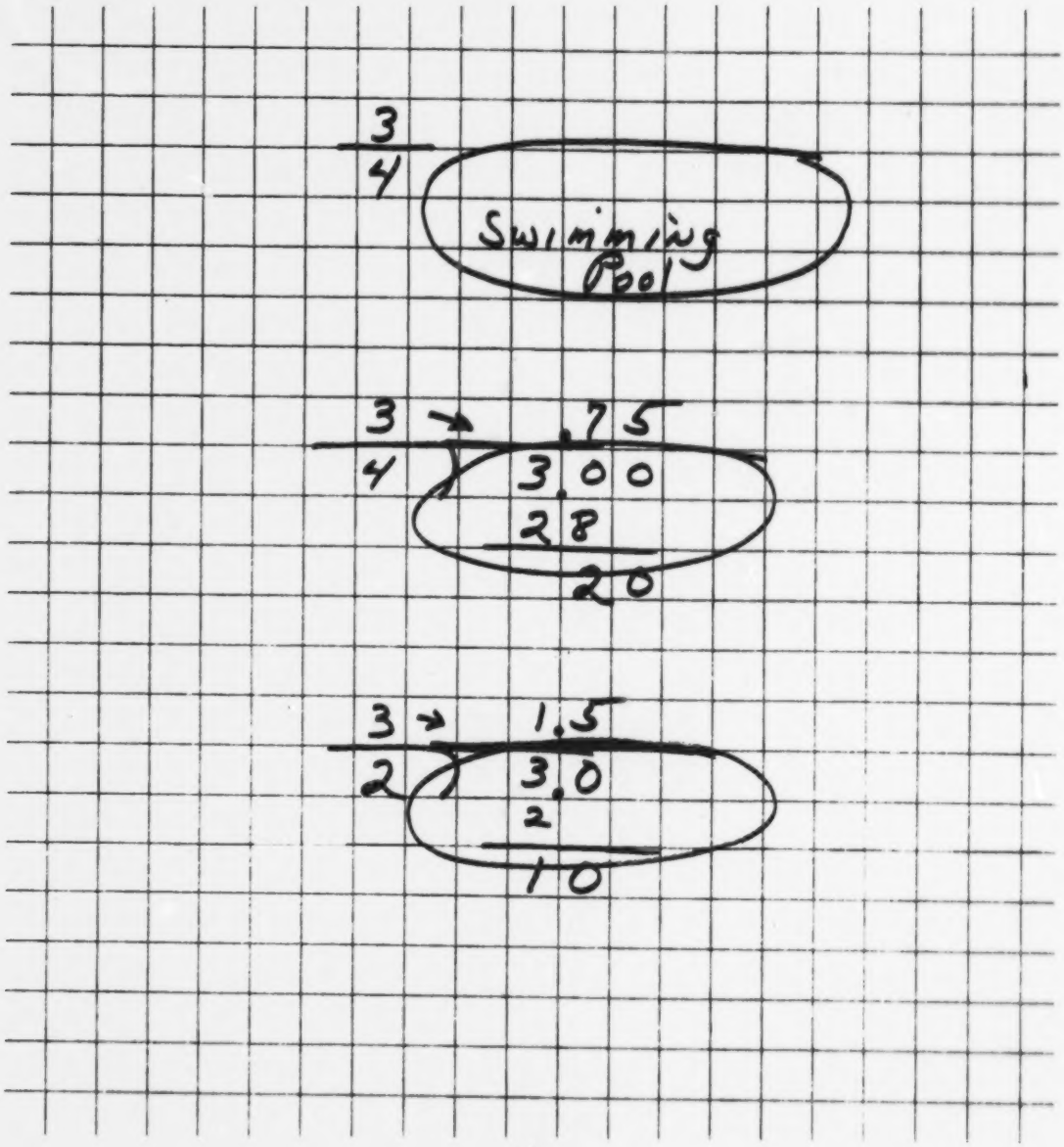
Even numbers: Students should learn to look for even numbers in fractions. When they encounter even numbers on both top and bottom of a fraction, they know it can be reduced.



Other Parts: Many students with weak math skills do not realize that fractions, decimals and percentages are ways to express the same values. They see these different ways to express parts as completely different and distinct. The students need to learn that each fraction has a decimal and percentage equivalent. Working the student through the following page chart can help them to understand this concept.

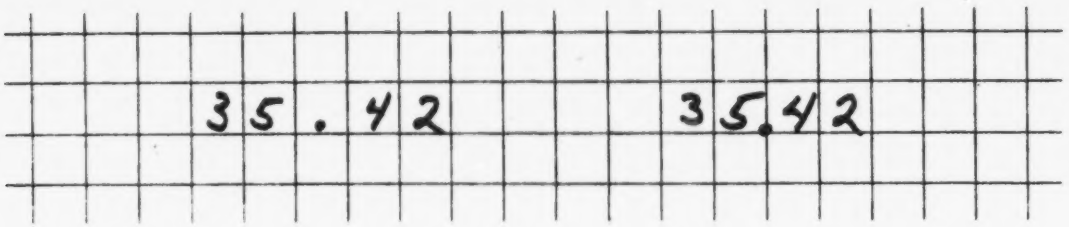
1	2/2	3/3	4/4	5/5	6/6	8/8	16/16	1.00	100%
							15/16	.9375	93.75%
						7/8	14/16	.875	87.5%
					5/6			.833	83.3%
							13/16	.8125	81.25%
				4/5				.80	80%
			3/4			6/8	12/16	.75	75%
							11/16	.6875	68.75%
		2/3			4/6			.666	66.6%
						5/8	10/16	.625	62.5%
				3/5				.60	60%
							9/16	.5625	56.25%
	1/2		2/4		3/6	4/8	8/16	.50	50%
							7/16	.4375	43.75%
				2/5				.40	40%
						3/8	6/16	.375	37.5%
		1/3			2/6			.333	33.3%
							5/16	.3125	31.25%
			1/4			2/8	4/16	.25	25%
				1/5				.20	20%
							3/16	.1875	18.75%
					1/6			.1666	16.66%
						1/8	2/16	.125	12.5%
							1/16	.0625	6.25%

Swimming Pool: A mnemonic technique which helps students with fractions is the swimming pool. Converting fractions to decimals is a simple procedure for students who can divide when they visualize a swimming pool. When a student wants a decimal equivalent, the student has the top number drop into the swimming pool which is then changed into a division bracket. See the following example.

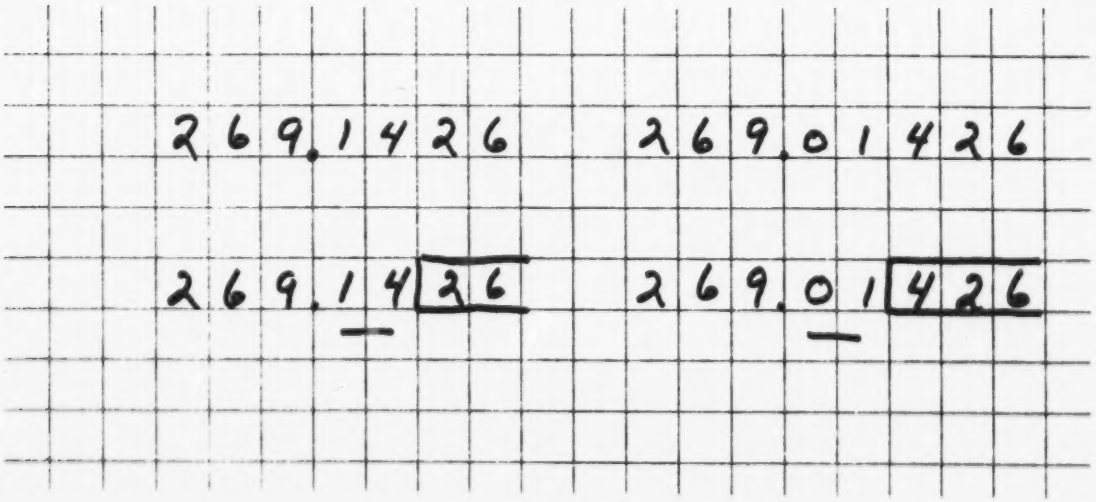


Decimals

As with other aspects of math, graph paper should be used to help students keep their numbers and decimal points in line. For students with visual processing problems, it is better for them to use a whole block for the decimal point. Others can place the decimal point on the line between the blocks.

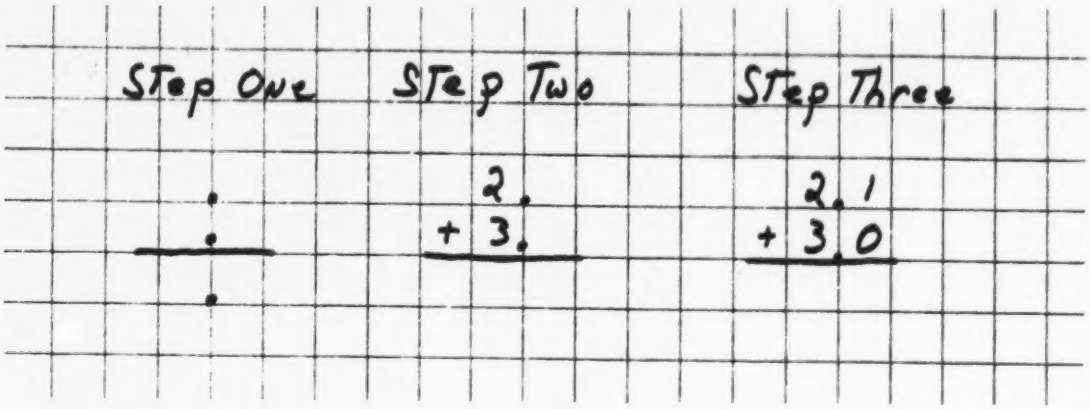


Another way to help students understand decimals is to have the students refer to money. A half dollar is the same as .50 and a quarter equals .25. Additionally, students can use the money reference to understand the value of numbers which have more than two number after the decimal point if they learn to cover all but the last two numbers to the right of the decimal point. This is also very helpful in teaching them the difference between .1 and .01. See example.

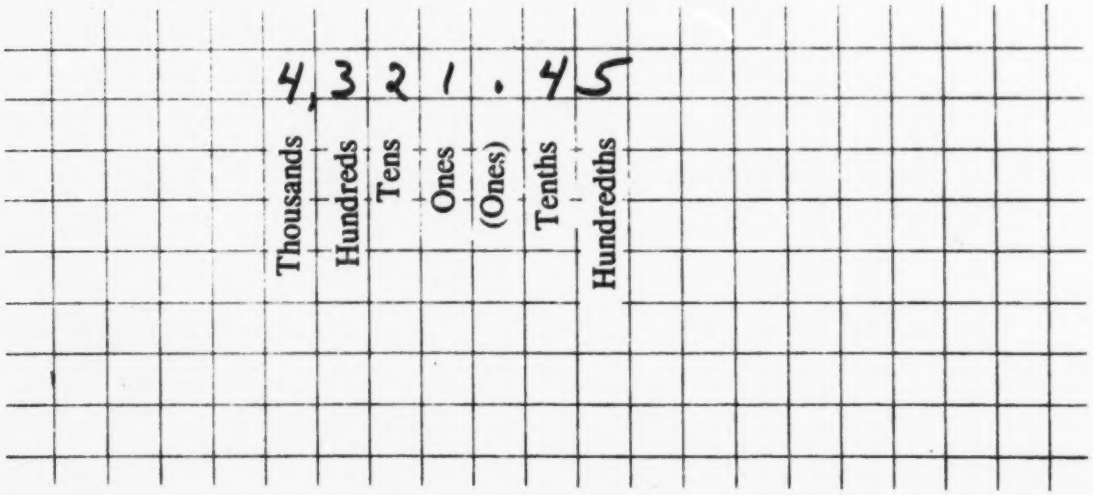


Since students with visual tracking problems have difficulty lining up numbers and decimal points, it is often helpful for them to place the decimal points first when writing numbers as demonstrated below.

2.1 + 3



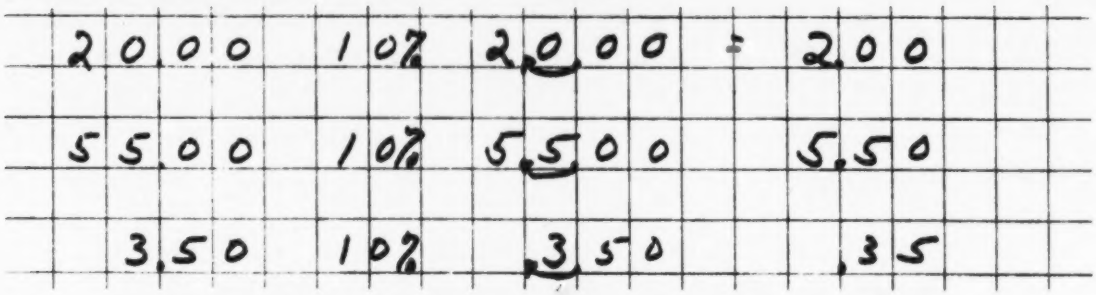
When teaching students the place values of numbers to the right of the decimal point, it is helpful to teach the student to read the decimal point as if it were the one's column. This makes it easier for the students to remember that the number after the decimal point is in the tenth place.



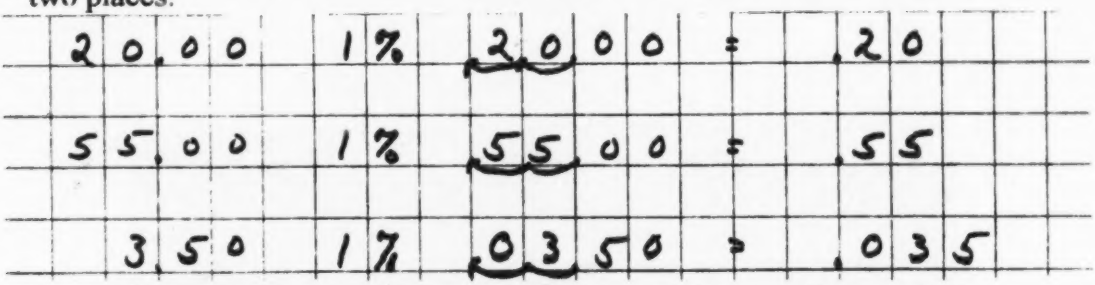
Percentages

Students who have a right/left discrimination problem often have difficulty learning how to calculate percentages. As with fractions, students may be able to remember how to use simple percentages (e.g. 10%) but they often have forgotten how to use other percentages and more complex operations involving percentages.

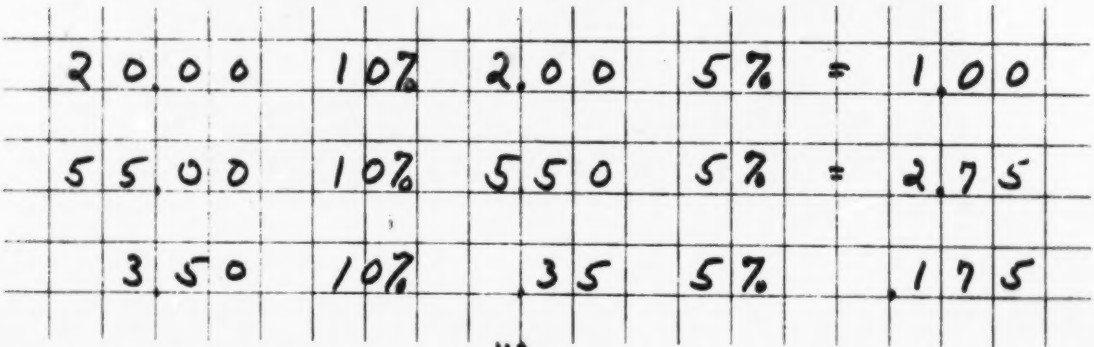
A simple way to enable the students to use percentages is to teach them how to convert percentages into decimals by moving the decimal point. First teach the student how to find 10% of numbers.



Next, teach the student how to find 1% by moving the decimal point two places.



The next step, uses the concept of halves which was discussed with fractions. The student is shown how to obtain 5% by dividing 10% in half. With these skills the student then can find the percentage of any number.



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Measuring

Students who do not understand fractions or decimals find measuring mysterious. The mystery can be reduced by teaching students to measure first with approximate measures, then gradually increasing the precision of their measurements. The student begins by measuring items in his/her environment and recording the measurements, rounding the measurements to the nearest inch. When the student can measure to the nearest inch with ease, the concept of the half inch can be introduced. The student then measures and records items to the nearest half inch. Next the concept of a quarter inch is introduced. This usually requires more instruction and practice. Again, when the student has mastered measuring to the nearest quarter inch, the concept of an eighth and, if necessary, a sixteenth and smaller, is introduced. I have found that as the fractions get smaller, the students will need more instruction and practice to master the measurement.

The following illustration demonstrates why some students find measuring difficult. Notice that the numbers are to the left of the inch mark. What is obvious to most people is a mystery to the student who does not understand fractions or measuring.

Numbers and Shapes

Many students with learning problems find geometry easy because it deals with objects which are concrete and visual. What many students find difficult is remembering the common numbers associated with shapes. This is because they do not use them enough. Too often math instruction is broken into units which isolates concepts and operations. This is particularly true with geometry. Students learn how to calculate perimeter, area and volume and then rarely use or review the operations after they move on to new material. Students can master geometry and remember the concepts better if it is reviewed periodically. There are a few basic geometric concepts and numbers which students should learn to the level of automatic recall as they can recall number facts. These include: 360 degrees in circles and quadrilaterals, 180 degrees in a triangle, 90 degrees in a right angle and half a 90 degree angle is 45 degrees. The diagram below shows these numbers.